

★ NATIONAL LEVEL ★

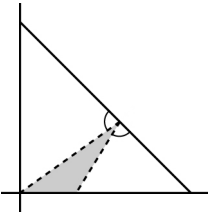
November 2009

The Mandelbrot Competition

Round One Test

Name: _____

Time Limit:
40 minutes

1. As part of an arithmetic drill, Jerry writes down a positive integer that does not end with a 0 or 5. He then also writes down the two nearest multiples of five (above and below his number) and adds together all three integers. If his total is 876, then what number did he originally choose?		①
2. Which of the following numbers is the largest: $\log_2 3$, $\log_3 5$ or $\log_5 11$?		①
3. Abigail is thinking of a positive integer m . She supplies the following four clues regarding her number. a) If $m > 20$ then m is divisible by 7. b) If $m < 40$ then m is 1 less than a multiple of 3. c) If $m > 60$ then m is prime. d) If $m < 80$ then m is a perfect square. What is Abigail's number?		②
4. How many positive integers from 1 to 2009 have the property that, when tripled, they give a result having all even digits?		②
5. Imagine that a mirror has been set up along the line $x + y = 12$. A laser beam emerging from the origin reflects off the mirror and hits the x -axis at the point $(4, 0)$, as shown in the diagram. What is the area of the region enclosed by the laser beam and the x -axis?		②
6. Let $\xi = \cos(\frac{2\pi}{7}) + i \sin(\frac{2\pi}{7})$ be a seventh root of unity. (Thus ξ is a certain complex number satisfying $\xi^7 = 1$.) Compute the value of $(2\xi + \xi^2)(2\xi^2 + \xi^4)(2\xi^3 + \xi^6)(2\xi^4 + \xi^8)(2\xi^5 + \xi^{10})(2\xi^6 + \xi^{12})$.		③
7. A red marker begins in the top left corner of a 1609×2009 grid of squares. It moves across the top row, down one square, back to the left along the second row, down one square, and so on. A green marker starts in the same place but moves down the first column, right one square, back up the second column, right one square, and so on. They each move one square at a time and thus reach the bottom right corner simultaneously. How many squares (including the starting and ending squares) do they occupy at the same time?		③

SCORE: