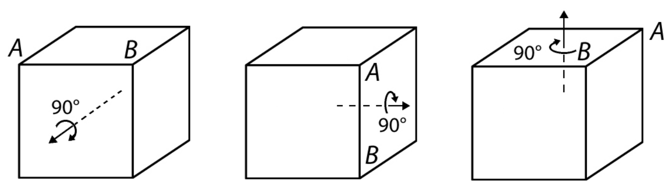
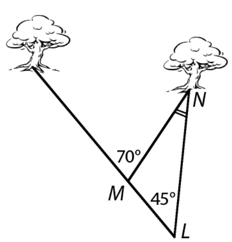
Answer Key	
1. 17	4. 13
2. 3	5. \$31
3. $25°$	6. 2017
	7. $2\sqrt{13}$

1. This linear equation is straight-forward to solve in the usual manner. However, it is also possible to solve the equation by inspection. Observe that because of the symmetry of the two sides of the equation, plugging in $x = 17$ gives $7 \cdot 10 = 70$ on both sides. Therefore $x = \mathbf{17}$ is the answer.

2. A sequence of three quarter-turns is the least number possible. There are several ways to accomplish this task; one way is illustrated below. After the third turn vertices A and B will have reversed positions.



3. Once we sketch an accurate diagram the geometry becomes fairly clear. Lars, Molly and Nate are located at points L, M and N, respectively. We are given the $45°$ and $70°$ angles and wish to find the measure of the marked angle. But the Exterior Angle Theorem states that the sum of the marked angle plus $45°$ equals $70°$, so the angle Nate measures is $\mathbf{25°}$.



4. We quickly discover that 14 is nimble, since $14 = 27 - 13$, and $27 = 3^3$ while 13 is prime. At first it would seem that 13 cannot be expressed as a cube minus a prime, but the persistent solver finally discovers that $13 = 512 - 499$, and $512 = 8^3$ while 499 is prime. Hence the next nimble number is in fact **13**. It seems likely that every positive integer is nimble, although a proof of this fact is probably not easy!

5. Let x represent the price Abby will charge for the new cheaper model, so that the more expensive type will cost $x + 8$ dollars. Also let w stand for the number of widgets sold, which does not change after the new pricing structure is put in place. Abby's revenue prior to the change is $\$30w$, while it becomes

$$\left(\frac{3}{4}w\right)x + \left(\frac{1}{4}w\right)(x+8) = w(x+2)$$

afterwards, since three-quarters of the sales are for $\$x$ each, while a quarter of them sell for $\$(x + 8)$. Since a 10% increase means that the new revenue is $\frac{110}{100}$ times the old revenue, we have

$$\frac{110}{100}(30w) = w(x+2) \qquad \Longrightarrow \qquad 33 = x + 2,$$

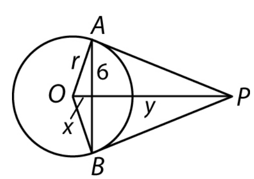
thus the less expensive widget should cost **\$31**.

6. The given expression may be rewritten as

$$\frac{1}{2010} - \frac{1}{m} = \frac{m - 2010}{2010m}.$$

As long as $m - 2010$ divides evenly into $2010m$ then the fraction will reduce to the form $\frac{1}{n}$. This certainly occurs when $m = 2011$, 2012, 2013, 2015 and 2016 since 2010 is divisible by 1, 2, 3, 5 and 6. The fraction also reduces when $m = 2014$, since both 2010 and m are even, hence $m - 2010 = 4$ will divide evenly into $2010m$. However, the fraction does not reduce for $m = 2017$, since neither m nor 2010 is divisible by 7, so **2017** is the smallest such number.

7. There are many routes to the answer. One method is note that $\overline{AB}$ is bisected by $\overline{OP}$ into two halves of length 6. On the other hand, $\overline{AB}$ cuts $\overline{OP}$ into two segments whose lengths we label as x and y. Next observe that $\triangle OAP$ is a right triangle since the line is tangent to the circle. Standard properties of altitudes in right triangles now imply that $xy = 6^2$. (Because the two



smaller right triangles within $\triangle OAP$ are similar.) But $x + y = 13$, and solving these equations yields $x = 4$ or 9. Hence $r = 2\sqrt{13}$ or $r = 3\sqrt{13}$. We discard the latter solution since $r < 10$, leaving $r = \mathbf{2\sqrt{13}}$.

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⋆ Regional Level ⋆

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