November 2011

⋆ National Level ⋆

The Mandelbrot Competition

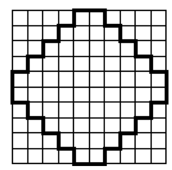
Round One Test

Time Limit: 40 minutes

Name: ______________________________

1. Consider the points with coordinates $(0, 2)$ and $(1, 1)$. How many circles pass through both of these points and are also tangent to the x-axis? 1

2. Write the numbers from 1 to 100 in the grid of squares shown here, going from left to right across each row, filling the rows from top to bottom. Next combine all the numbers within the highlighted region by adding together the even numbers, then subtracting the odd numbers. What is the total? 1

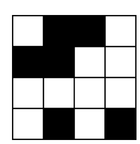


3. Draw any unbroken path from $(1, 1)$ to $(4, 4)$ in the Cartesian plane. Now check each point having integer coordinates, such as $(2, 3)$ or $(0, 4)$, and color it red if it lies within 0.9 units of some point on the path. What is the smallest possible number of red points one could obtain? 2

4. Find the smallest positive integer n for which the sum $14 + 15 + \cdots + n$ is equal to a power of 3. 2

5. Recall that the Fibonacci numbers are defined as $F_1 = 1$, $F_2 = 1$, and $F_{n+1} = F_n + F_{n-1}$ for all $n \geq 2$. Express $3F_{2012}^2 - 4F_{2011}F_{2010} - 3F_{2010}^2$ as a product of two Fibonacci numbers; i.e., write your answer in the form F_mF_n. 2

6. In how many ways is it possible to color the squares of a 4×4 grid either black or white so that every pair of rows has matching squares at exactly two of the four positions within the rows? One such configuration is shown at right. 3



7. Calculate the exact value of the infinite continued fraction at right. 3

$$1 + \cfrac{1}{2 + \cfrac{4}{2^2 + \cfrac{4^2}{2^3 + \cfrac{4^3}{2^4 + \cdots}}}}$$

SCORE: