



★ REGIONAL LEVEL ★

November 2018

The Mandelbrot Competition

Round Two Test

Name: _____

Time Limit:
40 minutes

1. A composite is a positive integer greater than 1 that is not a prime, such as 4 or 14. Let A be the product of the first five composite numbers, and let B be the product of the next five composite numbers. Compute B/A .	1
2. Using the given numbers, start a path at 1, move to one of its eight neighbors, then circle around in either direction until every number is visited once. The <i>weight</i> is the total of the positive differences between consecutive numbers in the path. Thus the path shown has weight 23. What is the minimum possible weight of a path with these numbers?	1
3. Rose and Karl walk along a 7 mile trail. Rose walks at 3 miles per hour, and Karl walks 2 miles per hour. Since Karl falls behind, Rose waits for him at each trail marker. The trail markers are located 1.7, 3.6, 5.5, and 7 miles into the trail. For how many minutes in total does Rose wait for Karl?	2
4. Let $ABCD$ be a square with side length 4 and let E be the point on \overline{AB} such that $BE = 1$. The circle centered at D passing through C intersects \overline{DE} at F inside $ABCD$. Compute the area of triangle BEF .	2
5. Suppose that α and β are measures of acute angles, with $0^\circ < \alpha < \beta < 90^\circ$. If $\sin \alpha = \cos \beta$ and $\tan(2\alpha) = \tan(3\beta)$, then find α , in degrees.	2
6. Suppose there is a $\frac{1}{3}$ probability that a person knows that cashews in fact come from a fruit. Given ten people at a party, how many pairs of people, on average, will there be who both know the truth concerning the cashew?	3
7. Evaluate the following sum, writing your answer as a reduced fraction. $\frac{100 + 101}{100^2 \cdot 101^2} + \frac{101 + 102}{101^2 \cdot 102^2} + \frac{102 + 103}{102^2 \cdot 103^2} + \cdots + \frac{199 + 200}{199^2 \cdot 200^2}$	3

SCORE: