March 2013

⋆ National Level ⋆

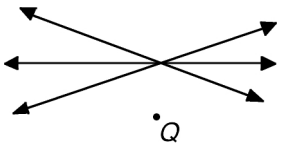
The Mandelbrot Competition

Round Five Test

Time Limit: 40 minutes

Name: _____

1. Given three lines that all cross at the same spot and a point Q below them, how many lines through Q intersect the rest of the diagram in fewer than three points?



(1)

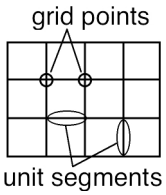
2. How many times larger than 5×10^{20} is 2×10^{50}? Write your answer in the form $C \times 10^k$ where $1 \leq C < 10$ and k is an integer.

(1)

3. Sutton has received three different test scores in his math class. It turns out that the average of his test scores is 6 more than his median (middle) test score. Let A be the difference between his higher two test scores, and let B be the difference between his lower two test scores. Determine $A - B$.

(2)

4. Draw a rectangle on a sheet of graph paper whose sides lie along the grid lines. We compute its "score" by awarding 7 points for each grid point enclosed and deducting 3 points for each unit segment inside. The rectangle shown encloses 6 grid points and 17 unit segments, so its score is $6(7) - 17(3) = -9$. Suppose that a certain rectangle has height and width both at least 8, and its score is 2013. What is its area?



(2)

5. Laura chooses a positive integer from 1 to 45 at random and lists all of its positive divisors. Then Ben selects one of these divisors, again at random. Which positive integer from 1 to 45 is least likely to occur as Ben's number?

(2)

6. For a positive real number x, let $\langle x \rangle$ represent the fractional part, meaning the portion after the decimal point. Thus $\langle \pi \rangle = .14159\ldots$, $\langle \frac{15}{4} \rangle = .75$ and $\langle 7 \rangle = 0$. Now define $f(x) = x + 100\langle x \rangle$ for $x \geq 0$. Find the average of all solutions to $f(x) = 2013$.

(3)

7. A triangle has sides of length 15, 21 and 24. It is possible to inscribe an ellipse within this triangle, tangent to each of the three sides at the midpoint of that side. Compute the area of this ellipse.

(3)

SCORE: