December 2018

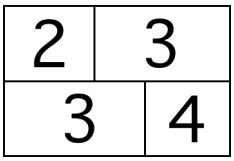
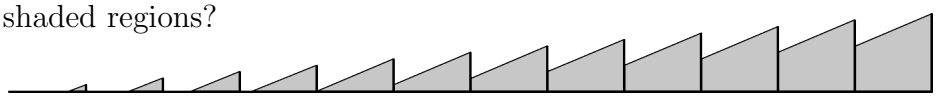
⋆ Regional Level ⋆

The Mandelbrot Competition

Round Three Test

Time Limit: 40 minutes

Name: _____

1. Decide whether the following statement is true or false: "If the diagonals of a quadrilateral are congruent and are perpendicular, then the quadrilateral must be a square." (Write 'True' or 'False' as your answer.)	(1)
2. Two of the digits from 0 to 9 are used to create a two-digit number that is equal to the sum of the other eight unused digits. Find this two-digit number.	(1)
3. How many ways are there to place a 1, 2, 3, 4 or 5 in each rectangle at right so that adjacent rectangles (whose perimeters overlap) have digits differing by at most 1? It is fine to use digits more than once, as shown. 	(2)
4. Find all positive real numbers x such that $5^x + 3 < 2^{2x+1}$, expressing your answer as an inequality.	(2)
5. In the diagram below the vertical segments are spaced 1 unit apart, are perpendicular to the base, and have heights of $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, $\ldots$, $\frac{12}{12}$ from left to right. If each slanted segment has slope $\frac{5}{12}$, then what is the total area of all the shaded regions? 	(2)
6. We say that a positive integer is *quirky* if dividing it into 1,000,000 gives the same quotient as remainder. For instance, 3000 is not quirky, since the quotient is 333 while the remainder is 1000, which are not the same. How many positive integers are quirky?	(3)
7. A lattice point is a point in the plane having integer coordinates. Suppose that a certain quadrilateral has all four vertices at lattice points, encloses eight lattice points in its interior, and has sides of length 11, $2\sqrt{10}$, $3\sqrt{13}$ and $4\sqrt{2}$ in some order. Determine its area.	(3)

SCORE: _____