



★ REGIONAL LEVEL ★

November 2009

# The Mandelbrot Competition

## Round One Test

Name: \_\_\_\_\_

*Time Limit:*  
40 minutes

1. As part of an arithmetic drill, Jerry writes down a positive integer that does not end with a 0 or 5. He then also writes down the two nearest multiples of five (above and below his number) and adds together all three integers. If his total is 876, then what number did he originally choose?		1
2. Let $P$ be a point in the plane, and let $l_1$ be a line through $P$ . Rotate this line about $P$ by $103^\circ$ clockwise to obtain line $l_2$ . Then rotate $l_2$ a total of $103^\circ$ clockwise about $P$ to get line $l_3$ . Repeat this process a total of five times, until lines $l_1$ through $l_6$ have been drawn. What is the smallest angle (in degrees) formed by any two of these lines?		1
3. Which of the following numbers is the largest: $\log_2 3$ , $\log_3 5$ or $\log_5 11$ ?		2
4. Abigail is thinking of a positive integer $m$ . She supplies the following four clues regarding her number. a) If $m > 20$ then $m$ is divisible by 7. b) If $m < 40$ then $m$ is 1 less than a multiple of 3. c) If $m > 60$ then $m$ is prime. d) If $m < 80$ then $m$ is a perfect square. What is Abigail's number?		2
5. Let $f(x)$ be a function satisfying the equation $f(f(x)) = 3x + 1$ . If we are told that $f(0) = 223$ , then what must be the value of $f(4)$ ?		2
6. How many positive integers from 1 to 2009 have the property that, when tripled, they give a result having all even digits? For example, 1534 is one such number since $3(1534) = 4602$ .		3
7. Imagine that a mirror is situated along the line $x + y = 12$ . A laser beam emerging from the origin reflects off the mirror, creating equal angles as shown, and hits the $x$ -axis at the point $(4, 0)$ . What is the area of the region enclosed by the laser beam and the $x$ -axis?		3

SCORE: