



★ NATIONAL LEVEL ★

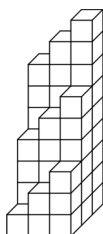
January 2018

The Mandelbrot Competition

Round Four Test

Name: _____

Time Limit:
40 minutes

<p>1. It is possible to place eight of the nine digits 1, 2, 3, 4, 5, 6, 7, 8, 9 into the squares below to create four true statements. Which digit is <i>not</i> used?</p> <p>$\square \div \square = 2$, $\square - \square = 4$, $\square \times \square = 6$, $\square + \square = 8$</p>		1
<p>2. A solid is built by stacking 45 unit cubes into nine columns on a desk, as shown. The front row of the solid has stacks of height 1, 2 and 3 unit cubes. The middle row has stacks of height 4, 5 and 6, while the back row has stacks of height 7, 8 and 9. What is the surface area of the resulting solid? (Include the bottom.)</p>		1
<p>3. Let p, q, r and s be prime numbers satisfying the equations below. What is the value of the largest of these four primes?</p> <p>$p + q + r = 72$, $p + r + s = 74$, $q + r + s = 89$</p>		2
<p>4. Let x and y be positive real numbers such that $\log_x y - \log_y(x^2) = 1$ and $x + y = 20$. Find the sum of all possible positive values of x.</p>		2
<p>5. Points X and Y along chord \overline{AB} of a circle are situated so that $AX = 3$, $XY = 4$ and $YB = 5$. If \overline{OX} and \overline{OY} are perpendicular, where O is the center of the circle, then find the area of this circle.</p>		2
<p>6. To <i>prune</i> a sequence a_1, a_2, a_3, \dots is to keep only the terms in positions $a_1, 2a_1, 3a_1$, and so forth. Beginning with the sequence 2, 3, 4, 5, \dots, repeatedly prune this sequence twenty times. In the resulting sequence, what is the remainder when the first term is divided by 41?</p>		3
<p>7. Let a, b and c be nonzero real numbers satisfying $a + b + c = abc$ and</p> $(a + b)(a + c)(b + c) = (a + b + c)^2 \left(\frac{a+1}{a} + \frac{b+1}{b} + \frac{c+1}{c} \right).$ <p>Determine the only possible value of $a + b + c$.</p>		3

SCORE: