March 2014

⋆ National Level ⋆

The Mandelbrot Competition

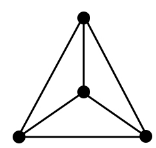
Round Five Test

Time Limit: 40 minutes

Name: ____________________________________

1. Determine the smallest real number satisfying $|x-20|+|x-14|=8$. (1)

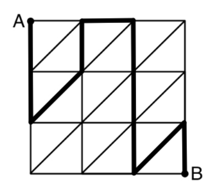
2. A marker is placed on each large dot in the diagram at right. Each of the four markers is red on one side and black on the reverse side; to begin all the red sides are facing up. On a move you swap any two markers and flip them both over. What is the least number of moves needed to return each marker to its starting dot, but have them all black side up instead? (1)



3. Find four integers such that their sum is -3 while the sum of their cubes is 3. (You may write the numbers in any order for your answer.) (2)

4. Find the largest six-digit number using each of the digits from 1 to 6 once such that this number is divisible by 6, deleting the 6 gives a number divisible by 5, deleting the 5 leaves a number divisible by 4, and so forth down to 1. (Thus deleting the 6 from 136245 gives 13245, then deleting the 5 gives 1324.) (2)

5. How many paths are there from A to B through the network shown if you may only move up, down, right, and up-right? A path also may not traverse any portion of the network more than once. A sample path is highlighted. (2)



6. Convex quadrilateral $ABCD$ has sides $AB=20$, $BC=14$, $CD=15$ and $DA=21$. There are many possible such quadrilaterals, but every one has an inscribed circle. What is the largest possible radius of such a circle? (3)

7. For $n \geq 1$ the nth triangular number is $T_n = 1+2+\cdots+n$. Define a sequence $a_1, a_2, a_3, \ldots$ to be the non-triangular positive integers in increasing order; namely 2, 4, 5, 7, 8, 9, 11, 12, 13, … Thus $a_1=2$, $a_2=4$, $a_3=5$, etc. Then let $b_1=a_{T_{100}}$, $b_2=a_{b_1}$, $b_3=a_{b_2}$, and so forth. Compute b_{100}. (3)

SCORE: