Answer Key	
1. $2\frac{1}{2}$	4. 6
2. $2\sqrt{5}$	5. $(3, -2, 2)$
3. B	6. 61
	7. 80

1. The tortoise walks 1 foot every 4 seconds, so he will need 2000 seconds to finish the 500 foot course. The hare will only take $500/10 = 50$ seconds to cover the same distance, but we must add in $60 \cdot 30 = 1800$ seconds for her nap. She still wins though, by $2000 - 1850 = 150$ seconds, which translates to $\mathbf{2\frac{1}{2}}$ minutes.

2. Observe that the tips of the shadow are cast by a pair of points at opposite corners of the rectangle. Thus the shadow will be longest when the diagonal is horizontal, at which time the shadow will be the same length as this diagonal. By the Pythagorean Theorem, this length is $\sqrt{2^2 + 4^2} = \mathbf{2\sqrt{5}}$.

3. One approach is to employ the quadratic formula to compute the positive roots of each function, then compare them directly. One should find that the positive root of the second quadratic, namely $-2 + \sqrt{13}$, is the largest. Here is an alternate approach. The graph of each function is a parabola. Note that decreasing the constant term lowers the parabola, which has the effect of spacing the roots out, so the positive root becomes larger. Similarly, decreasing the linear coefficient (from $5x$ to $4x$) has the effect of moving the parabola to the right, which also increases the roots. Thus the parabola with the smallest coefficients has the largest positive root, hence $\boldsymbol{B}$ is the correct answer.

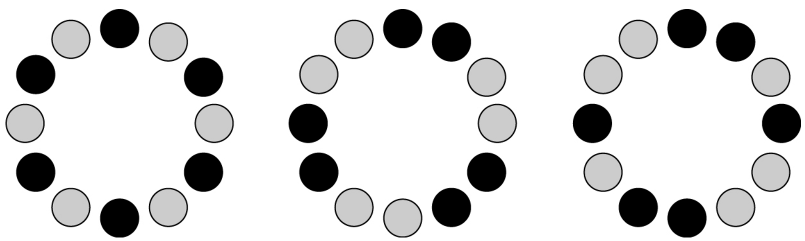
4. Suppose we don't make any moves of size 25. Then the best we can do is to move five 11's to the right followed by three 18's to the left, which is eight moves. If we move one 25 to the right, it takes at least eleven more moves to reach the asterisk, but if we move one 25 to the left, followed by an 18 to the left and four 11's to the right, we land on the asterisk in only six moves. The same can be accomplished by moving two 25's to the left, then three 11's and one 18 to the right, but all other combinations of moves involve more steps, so **6** is the best possible.

5. Perhaps the most efficient and entertaining route to the answer is to systematically search for the fourth vertex via trial and error. Using the distance formula we determine that the sides of the regular tetrahedron have length $\sqrt{18}$. So to reach the fourth vertex we may begin at any of the given vertices and move ± 4, ± 1 and ± 1 units in the x, y and z-directions in any order, or move ± 3, ± 3 and 0 units instead.

For instance, suppose we begin at $(0, 1, 2)$ and add or subtract 4, 1 and 1 to the coordinates in some order. We can't subtract the 4 from any of the coordinates, since the resulting location would be too far from the other points. Similarly, we can't add 4 to the z-coordinate either, since the point $(*, *, 6)$ is too far from $(4, 2, 1)$. But if we add 4 to the x-coordinate then we must add or subtract 1 from the y-coordinate, and the resulting points $(4, 2, *)$ or $(4, 0, *)$ cannot be $\sqrt{18}$ away from $(4, 2, 1)$ in either case. Continuing in this manner we eliminate all possibilities involving 4, 1, 1. We eventually discover that adding 3, -3, and 0 to the coordinates of $(0, 1, 2)$ gives $\mathbf{(3, -2, 2)}$, which is a distance of $\sqrt{18}$ from all three given points, as desired.

6. In order to solve this problem we need to find the remainder that 2^n leaves when divided by 7. For $n = 0$ to 6 these remainders are 1, 2, 4, 1, 2, 4, 1, ..., repeating every three terms. (We leave it to the reader to learn the technique for finding such a pattern of remainders.) Hence $2^n - 2$ will be divisible by 7 exactly when n is 1 more than a multiple of 3. A similar analysis shows that $4^n - 4$ is divisible by 9 for the same values of n. Meanwhile, $6^n - 6$ is divisible by 11 precisely when n is 1 more than a multiple of 10, and $8^n - 8$ is divisible by 13 exactly when n is 1 more than a multiple of 4. Putting all these facts together, we find that n must be 1 more than a multiple of 60. The smallest such value of n is 1; the next smallest value is **61**.

7. There are $\binom{12}{6} = 924$ ways to choose six of the twelve spots to put the black chairs. (The silver chairs will go in the remaining spots.) It then seems reasonable to divide this quantity by 12—each arrangement appears to be counted twelve times since each configuration of black and silver chairs can be rotated to give a total of 12 arrangements which should all be counted as the same one. In actuality this is usually, but not always, the case. For instance, due to its symmetry the configuration on the left below only appears twice in the 924 possible arrangements, while the configuration in the middle only appears four times.



Finally, a configuration such as the one on the right, in which every pair of opposite chairs are the same color, will appear only six (rather than twelve) times among the 924 possible arrangements. There are $\binom{6}{3} = 20$ ways to color three of the six pairs black, but we must then subtract the two arrangements already counted by the left-hand case, for a total of 18 such arrangements which should count as only $\frac{18}{6} = 3$ different ways to place the chairs.

Hence there are 24 "special" arrangements which are overcounted less than twelve times due to their symmetry. These 24 arrangements account for a total of 5 different ways to place chairs. The remaining 900 arrangements contribute $\frac{900}{12} = 75$ ways, giving **80** different ways in total to place the chairs around the table.

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