



★ NATIONAL LEVEL ★

March 2010

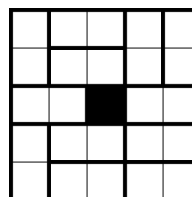
The Mandelbrot Competition

Round Five Test

Name: _____

Time Limit:
40 minutes

1. Suppose that $ABCD$ is an arbitrary rectangle. Let M be the midpoint of \overline{AB} and let N be the midpoint of \overline{AD} . Then \overline{MN} intersects diagonal \overline{AC} at a point P . Determine the value of the ratio AP/PC .	1
2. Mr. Strump has three daughters with integer ages. One of them is currently twice the age of another. One year from now it will again be the case that one of the girls is twice the age of another. Four years after that the same thing will occur once more. How old is the eldest daughter currently?	1
3. How many ways are there to write 84,000 as the product of two relatively prime positive integers? The order of the two factors does not matter; thus $8 \cdot 13$ and $13 \cdot 8$ would not be different ways of writing 104 as a product.	2
4. For a positive integer n , we define $g(n)$ to equal n plus the sum of the digits of n . Thus $g(25) = 25 + 2 + 5 = 32$, for example. Find the smallest positive integer n such that $g(n) = g(N)$ for some larger integer $N > n$.	2
5. Let θ be the angle in the first quadrant for which $4 \cos(2\theta) + 3 \sin(2\theta) = \frac{41}{16}$. Compute the value of $3 \cos \theta + \sin \theta$.	2
6. Consider a 5×5 grid of unit squares which is missing its center square. How many ways are there to tile this region with twelve 1×2 dominoes? One possibility is illustrated at right.	3
7. Find a quadratic function $f(x) = x^2 + ax + b$ such that $\frac{f(f(x) + x)}{f(x)} = x^2 + 1776x + 2010.$	3



SCORE: