November 2017

⋆ NATIONAL LEVEL ⋆

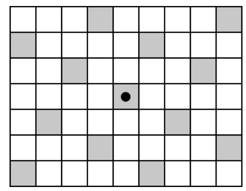
The Mandelbrot Competition

Round Two Test

Time Limit: 40 minutes

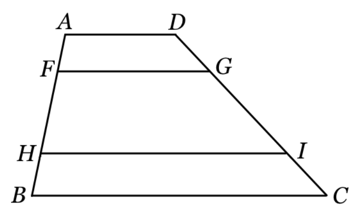
Name: ______________________________

1. Suppose this repeating pattern of shaded squares is extended infinitely in all directions, then the shaded square with the dot is erased. How many other shaded squares are visible from the center of the dot? One cannot see beyond any part of a shaded square, including its corner. (1)



2. Starting with the number 0, perform one of the following operations on each turn: add 2, subtract 2, or multiply by 2. What is the minimum number of turns required to obtain the number 60? (1)

3. Suppose $ABCD$ is a trapezoid with $\overline{AD} \parallel \overline{BC}$. Draw $\overline{FG}$ parallel to $\overline{AD}$ with $AF = \frac{1}{4}AB$, then draw $\overline{HI}$ parallel to $\overline{AD}$ with $AH = \frac{3}{4}AB$. If the perimeter of $AFGD$ is 10 while the perimeter of $AHID$ is 14, what is the perimeter of $ABCD$? (2)



4. Let r and s be real numbers from 0 to 1 chosen independently at random. What is the probability that at least one of the numbers $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ is located between r and s? (2)

5. Given the three equations $z^{60} = 1$, $z^{72} = 1$ and $z^{90} = 1$, how many complex numbers z are solutions to two of the equations, but not all three? (2)

6. Compute the remainder when P^2 is divided by 2017, where

$$P = \left(\sqrt{2} + \frac{1}{\sqrt{2}}\right)\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) \cdots \left(\sqrt{2015} + \frac{1}{\sqrt{2015}}\right).$$

(3)

7. Let S be the set containing all positive integers less than 132 that are not multiples of 12. Suppose S is partitioned into forty sets, each containing three of the numbers, along with one set containing the single leftover number. What is the largest possible value for the single leftover number, if each set of three numbers forms an arithmetic sequence with difference 1 or 12? (3)

SCORE: ______