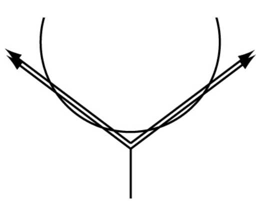
Answer Key	
	4. 12
1. 25	5. 38
2. 9	6. 6
3. 8	7. 12

1. In five hours Warren can build 20 puzzles, which requires exactly two boards worth of wood. Hence he spends \$26 on materials. On the other hand, the 20 puzzles net him 20(\$7.50) = \$150 in income. His profit is therefore \$150 − \$26 = \$124 in total, which comes to $\frac{1}{5}$(\$124) ≈ **\$25** per hour, to the nearest dollar.

2. We are interested in letters such as R, since R does not appear in the word INFINITESIMAL, but doesn't not appear in both of them. (Because it does appear in GARGANTUAN.) A letter such as V is no good, because V does not appear in the first word but it also does not appear in the second word. A moment's thought reveals that the question is actually just asking for those letters in one word or the other but not in both, albeit in a somewhat disguised manner. Those letters are R,U,G,S,M,I,L,E,F, for a total of **9** letters.

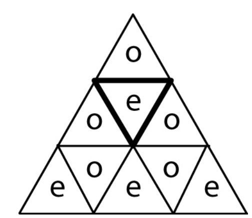
3. Any circle that encloses the common endpoints of the rays (so as to intersect all five rays at least once) can do no better than to create five points of intersection. However, by ignoring the lower ray and intersecting each of the others twice, as shown in the diagram at left, it is possible to obtain as many as **8** points of intersection between a circle and this object.



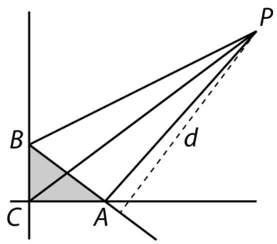
4. It is fairly clear that in order to prevent adjacent letters from appearing in the same row that we will need to place A, C, E in one row and B, D, F in the other. We have two choices for which row will take the letters A, C, E, and then there are six possible orders in which to list these three letters across the row. (Namely ACE, AEC, CAE, CEA, EAC and ECA.) We claim that at this point the remaining three letters can be position in exactly one way. For the D cannot be placed in the same column as either the C or E, hence it goes with the A. This forces the F to go with the C and the B to go in the same column as the E. We conclude that there are $2 \cdot 6 = \mathbf{12}$ possible ways to fill in the grid.

5. It is possible to rule out many values of n by observing that if either n or $n+1$ were a prime then the sum of the primes dividing these two numbers would be at least n, which is too large. So the first few pairs we need consider are $(8,9)$, $(9,10)$, $(14,15)$ and $(15,16)$. The sums of the primes dividing them are 5, 10, 17, 10 respectively, none of which equals $n-1$. But from this point on the sum will be too small unless n and $n+1$ are of the form $2p$ and $3q$ for primes p and q. So we proceed to check $(21,22)$, $(33,34)$, $(38,39)$ and $(57,58)$. The sums in these cases are 23, 33, 37, 53 respectively. Apparently $n = \mathbf{38}$ satisfies the statement. (The sum is too small for all $n > 38$, which is why the answer is unique.)

6. Since there are many more multiples of 2 than multiples of 7 available to us, it makes sense to begin by trying to arrange the five odd digits and the four even digits so as to create as many even three-in-a-row sums as possible. There is no way to arrange for all twelve such sums to be even (why not?), but we can come pretty close using the arrangement shown at right, in which 'e' represents an even digit and 'o' represents an odd digit. The only odd sums here are the lower left and lower right horizontal three-in-a-row sums, which therefore must be odd multiples of 7. Such sums can be obtained using $4+2+1$, $8+7+6$, $9+7+5$ and $9+8+4$. We must choose two sums which have a digit in common, so our only option is to use the digits 8, 9, 4, 1, 2 in that order (or the reverse order) across the bottom. Either way, the only remaining even digit is **6**, which will appear in the highlighted triangle.



7. The key to finding a clean method for computing the given quantity is to realize that the expression $5d$ is related to an area in the diagram. Labelling the points as shown, we first note that the line $\frac{1}{4}x + \frac{1}{3}y = 1$ has an x-intercept of 4 and a y-intercept of 3. In other words, $AC = 4$ and $BC = 3$. Since $\triangle ABC$ is a right triangle, this means that $AB = 5$.



We are now ready to demonstrate that $5d$ measures twice the area of $\triangle PAB$. (Note that the diagram is qualitatively correct, but is not drawn to scale.) The distance d from P to the line gives the height from P to base $\overline{AB}$, and $AB = 5$, thus $area(PAB) = \frac{1}{2}(5)(d)$, from which the claim follows. In the same manner we see that $3(2009)$ is equal to twice the area of $\triangle PBC$ and that $4(2010)$ is twice the area of $\triangle PAC$. In summary we have

$$\begin{aligned} 3(2009) + 4(2010) - 5d &= 2(area(PBC) + area(PAC) - area(PAB)) \\ &= 2\,area(ABC) \\ &= \mathbf{12}. \end{aligned}$$

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