



★ NATIONAL LEVEL ★

January 2013

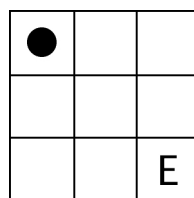
The Mandelbrot Competition

Round Three Test

Name: _____

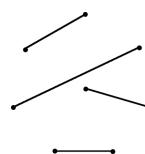
Time Limit:
40 minutes

1. Beginning with a score of 1, the object of this puzzle is to move the black marker to the bottom right square labeled E, ending with a score of 25. Each move to the right adds 2, each move to the left subtracts 1, each move down triples your score, while each move up halves the score. What is the least number of moves necessary to solve the puzzle?



1

2. The diagram given shows the vertices of a regular heptagon along with its center and illustrates one way to draw four non-intersecting segments using all eight points as endpoints. How many ways in total are there to draw four such segments?



1

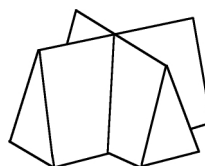
3. Philip chooses a point at random within a circle of radius five, then draws another circle with radius three having his chosen point as its center. Compute the probability that these two circles intersect one another.

2

4. It is a fact that $\frac{3}{17} = .1764705882352941176 \dots$. Create a new real number by writing a decimal point, then use the 3^{k-1} th digit past the decimal of $\frac{3}{17}$ as the k th digit past the decimal of the new number. What is the value of this new real number, written as a fraction in lowest terms?

2

5. A cabin consists of two triangular prisms meeting at right angles. Each prism has a base of width 10 feet, slant height (along the roof) of 13 feet on each side, and a length of 30 feet. Find the total volume of this structure in ft^3 .



2

6. Let $a_1 = \sqrt{3}$, $a_2 = 4 - \sqrt{3}$, $a_3 = 16 - 7\sqrt{3}$, and in general define the next term as $a_{n+1} = a_n^2 - a_n + 1$. Determine the value of $\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots$.

3

7. Let $\xi = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$ be a complex fifth root of unity. Set $a = 20\xi^2 + 13\xi$, $b = 20\xi^4 + 13\xi^2$, $c = 20\xi^3 + 13\xi^4$, and $d = 20\xi + 13\xi^3$. Find $a^3 + b^3 + c^3 + d^3$.

3

SCORE: