Answer Key	
1. 8	4. 21/25
2. 5	5. 17/101
3. 21	6. 3200
	7. 36

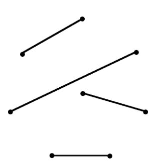
1. One productive way to find a route is to work backwards from the ending position. We can't have moved downwards as our final move, since 25 is not a multiple of 3, so we must have moved to the right with a score of 23. By the same reasoning the move before that must also have been to the right, and so forth. One route involving eight moves is right, left, down, right, left, down, right, right with scores

$$1 \to 3 \to 2 \to 6 \to 8 \to 7 \to 21 \to 23 \to 25.$$

No route uses fewer moves, so **8** moves is the answer.

2. To obtain as steep a slope as possible we should use the point furthest down and to the right in the first rectangle joined to the left/uppermost point in the other rectangle. The slope of this line is $(6-1)/(4-3) = \mathbf{5}$.

3. We first note that the center point can be joined to any of the vertices of the heptagon, giving 7 ways to begin drawing the segments. For each such segment there are precisely three ways to join the remaining six vertices. One way is shown at left, the second is the mirror image over the segment extending from the center, and the third way involves joining adjacent vertices in pairs around the perimeter. Hence there are a total of $7 \cdot 3 = \mathbf{21}$ ways.



4. A quick sketch reveals that the issue of whether the two circles intersect depends completely upon the distance between their centers. If the point used as the center of the circle with radius 3 is at least 2 units away from the center of the circle with radius 5, then the circles will intersect. Such points comprise the region within the circle of radius 5 but outside a circle of radius 2 having the same center. Hence the area of this region is $25\pi - 4\pi = 21\pi$, leading to a probability of $21\pi/25\pi = \mathbf{21/25}$.

5. We know that the decimal expansion for $\frac{3}{17}$ will begin repeating after at most 16 digits. A careful inspection of the given decimal shows that we do need the full 16 digits. We are now instructed to take digits in places 1, 3, 9, 27, 81, 243, ... to create a new decimal. Since every 16^{th} digit is the same, this amounts to taking digits 1, 3, 9, 11, 1, 3, ... from the decimal expansion. But as has now become evident, the powers of 3 repeat with period four. Therefore the desired decimal expansion is

$$.168316831683\ldots = \frac{1683}{9999} = \frac{\mathbf{17}}{\mathbf{101}}.$$

The fact that the final fraction reduces so much is intriguing. (But the 17 in the numerator is probably a coincidence.) Students with sufficient number theory background might consider looking for further examples along these lines. Please share any interesting discoveries with us at `info@mandelbrot.org`.

6. Several slippery computations can be avoided by adopting the strategy of adding together the volumes of two complete triangular prisms, then subtracting out the volume of the square-based pyramid in which they intersect. Recalling the 5–12–13 right triangle, we quickly deduce that the height of the structure is 12 feet. Hence the overall volume is

$$\tfrac{1}{2}(10)(12)(30) + \tfrac{1}{2}(10)(12)(30) - \tfrac{1}{3}(10)(10)(12) = 3600 - 400 = \mathbf{3200}.$$

7. Let us call the three mystery numbers a, b and c. According to Truelian it is the case that

$$\begin{aligned} ab - c &= 4, \\ ac - b &= 172, \\ bc - a &= 283. \end{aligned}$$

Notice that adding the first two equations yields

$$ab + ac - b - c = 176 \quad \Longrightarrow \quad (a-1)(b+c) = 2^4 \cdot 11.$$

By adding the other pairs of equations we also find

$$(b-1)(a+c) = 287 = 7 \cdot 41, \qquad (c-1)(a+b) = 455 = 5 \cdot 7 \cdot 13.$$

The second factorization is particularly useful. We can't have $a+c=1$, and it's fairly easy to rule out $b=2$, so we must have either $b-1=7$ and $a+c=42$ or $b-1=42$ and $a+c=7$. Plugging $b=43$ into the original equations leads nowhere, but trying $b=8$ gives the values $a=5$, $b=8$ and $c=36$, of which the largest is **36**.

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