



★ REGIONAL LEVEL ★

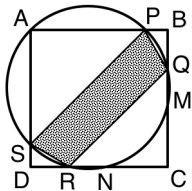
February 2014

# The Mandelbrot Competition

## Round Four Test

Name: \_\_\_\_\_

Time Limit:  
40 minutes

1. Vincent owes Melissa \$14, but as he hands her a \$20 bill they drop it down a drain and it falls beyond their reach. Assuming they share the loss equally, how much does Vincent owe Melissa now?		1
2. Drake, James and Sarah are each thinking of a different two-digit number. Each person has a different tens digit, Drake's number is the smallest, and the product of their three numbers is less than 100,000. What is the largest possible value for Drake's number?		1
3. The vertices of a certain square are located on the lines $y = 1$ , $y = 2$ , $y = 3$ and $y = 4$ , one vertex per line. What is the area of the square?		2
4. In the game of Frozzle you select one of the numbers $1, 2, \dots, 8$ in advance. The other player then picks one of these eight numbers at random, multiplies it with your number, takes the units digit, and gives you this amount. What number should you choose in advance to maximize your expected winnings?		2
5. At a certain party there are seven individuals who each know precisely seven other people at the party, while the remaining guests each know exactly five other people. (Knowing is mutual, meaning that if one person knows another, then that second person also knows the first.) What is the least number of people that could be at the party?		2
6. Given square $ABCD$ with sides of length 2, let $M$ and $N$ denote the midpoints of sides $\overline{BC}$ and $\overline{CD}$ . Suppose the circle through $A$ , $M$ and $N$ intersects the sides again at points $P$ , $Q$ , $R$ and $S$ , as shown. Compute the area of $PQRS$ .		3
7. For positive integers $n \geq 2$ , define $g(n)$ to be one more than the largest proper divisor of $n$ . Hence $g(35) = 8$ , since the proper divisors of 35 are 1, 5 and 7. For how many $n$ in the range $2 \leq n \leq 100$ do we have $g(g(n)) = 2$ ?		3

SCORE: