Answer Key	
1. $7/10$	4. $63°$
2. 7	5. 4000
3. -3	6. 812
	7. $(7,24)$ $(8,15)$ $(9,12)$

1. Since two-thirds of the students are girls, and one quarter of them are asleep, this accounts for $(\frac{2}{3})(\frac{1}{4}) = \frac{1}{6}$ of the students. The remaining one-third of the students are boys, of whom two-fifths are asleep, for another $(\frac{1}{3})(\frac{2}{5}) = \frac{2}{15}$ of the students. Hence a total of $\frac{1}{6} + \frac{2}{15} = \frac{9}{30} = \frac{3}{10}$ of the students are asleep, so $\mathbf{7/10}$ of them are awake. (How could anyone ever fall asleep during an event as exciting as a math class?)

2. When we add up the chips along each line (two horizontal and two vertical), we obtain a total of $(4)(5) = 20$ chips. Notice that each chip on a corner gets counted twice in this process, while the remaining chips are counted once. Or put another way, all 13 chips are counted once, then the chips on the corners are counted once more to bring the total up to 20, so clearly there must be **7** chips on the corners.

3. We confirm that $x = 1$ is a solution since $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$. To find the other solution we could either guess negative integers until we find one that works (probably the fastest approach!) or multiply through by the product $(x+1)(x+2)(x+5)$ to obtain

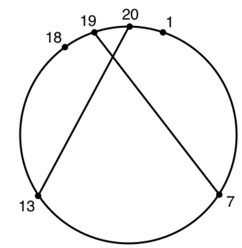
$$(x+2)(x+5) - (x+1)(x+5) = (x+1)(x+2).$$

Expanding the products yields

$$(x^2 + 7x + 10) - (x^2 + 6x + 5) = (x^2 + 3x + 2),$$

or just $x^2 + 2x - 3 = 0$, moving all terms to the right side. Factoring gives $(x-1)(x+3) = 0$, so the other solution is $x = \mathbf{-3}$.

4. A convenient means of keeping track of angles in a regular 20-sided polygon is via its circumscribed circle, since the twenty vertices divide the circumference into twenty congruent arcs, each measuring $18°$. Lines $P_{20}P_{13}$ and $P_{19}P_7$ subtend arcs $P_{20}P_{19}$ and $P_{13}P_7$, which measure $18°$ and $6(18) = 108°$, respectively. According to a theorem on measures of angles formed by intersecting chords, the acute angle formed by these lines measures $\frac{1}{2}(18° + 108°) = \mathbf{63°}$.



5. Each of the congruent rectangles will be quite long and skinny, meaning that practically the entire perimeter will be taken up by the length, as opposed to the height. In other words, each rectangle has a width that is very close to $\frac{1}{2}(2013) = 1006.5$, which is the width of the original square. Therefore the square has a perimeter of about 4026, so we predict that its perimeter is **4000**, when rounded to the nearest hundred. (In case you were wondering, the actual perimeter is about 4022.)

6. We organize our count by the location of the vertex V of the isosceles triangle. There are 42 ways to place this vertex, leaving twenty pairs of vertices that could serve as the endpoints of the base. (I.e. the vertices on either side of V, or the two vertices just beyond these, and so forth.) This would seem to give a total of $(42)(20) = 840$ isosceles triangles, but we have counted the $\frac{1}{3}(42) = 14$ equilateral triangles three times each, when they should only count once. Hence we subtract off 28 triangles to obtain a total of $840 - 28 = \mathbf{812}$ isosceles triangles.

7. We require the positive integers a, b and $\frac{1}{3}ab - a - b$ to satisfy the Pythagorean Theorem, which means that we need

$$a^2 + b^2 = (\tfrac{1}{3}ab - a - b)^2.$$

Expanding the right hand side gives

$$a^2 + b^2 = \tfrac{1}{9}a^2b^2 + a^2 + b^2 - \tfrac{2}{3}a^2b - \tfrac{2}{3}ab^2 + 2ab.$$

Cancelling the $a^2 + b^2$ on each side and dividing through by ab leads to

$$0 = \frac{1}{9}ab - \frac{2}{3}a - \frac{2}{3}b + 2.$$

Multiplying by 9, adding 18, and factoring yields

$$18 = ab - 6a - 6b + 36 = (a-6)(b-6).$$

We can now read off the pairs (a, b), since $(a-6)$ and $(b-6)$ must be positive integers multiplying to 18, such as $1 \cdot 18$, $2 \cdot 9$, or $3 \cdot 6$. The complete solution set is $(\mathbf{7}, \mathbf{24})$, $(\mathbf{8}, \mathbf{15})$, and $(\mathbf{9}, \mathbf{12})$.

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⋆ Regional Level ⋆

The Mandelbrot Competition

Round Two Solutions

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