ANSWER KEY		4.	neither
1.	789	5.	2/3
2.	25	6.	$4\sqrt{5}$
3.	10	7.	29952

1. One could simply list all the two-digit numbers containing the digit 3 and add them up, of course. Here is a shortcut, based on the fact that $1+2+3+\cdots+9=45$. We find that

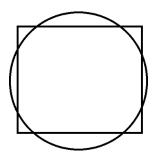
$$\begin{aligned}13+23+\cdots+93 &= (10+20+\cdots+90)+(3+3+\cdots+3)\\ &= 450+27 \;=\; 477.\end{aligned}$$

Similarly,

$$30+31+\cdots+39=300+45=345.$$

We counted 33 twice, so our total is $477+345-33=\mathbf{789}$.

2. Apparently the people in front of and behind Sonia account for a total of $32\%+64\%=96\%$ of the people in the line. Hence Sonia represents 4% of the people. But $4\%=1/25$, so we deduce that there must be **25** people in line.



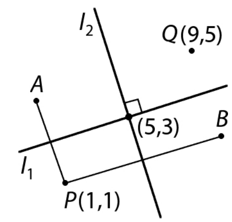
3. To increase the number of regions formed by the circle and rectangle we should increase the number of points at which they intersect. The circle may intersect each side of the rectangle at most twice, so we can obtain a maximum of **10** regions using a circle and a rectangle, as shown.

4. It is in fact true that a line with negative slope must pass through quadrant *II*. However, a line such as $y=-3$ with *zero* slope does not have to pass through quadrant *II*. Hence the slope m is not necessarily positive. In the same manner, the line $y=x$ has a y-intercept of 0 (which is not negative) but does not pass through quadrant *II*. Therefore **neither** of the given statements must be true.

5. It stands to reason that in order to make a and d small, we should choose large values for b and c, then take $ab=1$, $bc=9$ and $cd=1$. By experimenting along these lines, it is not hard to come upon the correct answer. A rigorous explanation requires the use of the AM-GM inequality, which states that $\frac{1}{2}(x+y)\ge\sqrt{xy}$ for positive real numbers x and y. Using $x=a$ and $y=d$ yields

$$\frac{a+d}{2}\ge\sqrt{ad}=\sqrt{\frac{ab\cdot cd}{bc}}\ge\sqrt{\frac{1\cdot 1}{9}}=\frac{1}{3}.$$

Hence $a+d\ge\mathbf{\frac{2}{3}}$, which can be achieved by taking $a=\frac{1}{3}$, $b=3$, $c=3$ and $d=\frac{1}{3}$.



6. Observe that line l_2 has an angle of inclination exactly 90° more than line l_1, which implies that l_2 is perpendicular to l_1, as suggested by the diagram at right. But segment $\overline{PA}$ is also perpendicular to l_1, since P reflects across line l_1 to A. Therefore $\overline{PA}\parallel l_2$, and similarly $\overline{PB}\parallel l_1$. The upshot is that P, A, and B are three vertices of a rectangle. Reflecting $P(1,1)$ over the point of intersection at $(5,3)$ gives the fourth vertex at $(9,5)$, which we label Q. Because the diagonals of a rectangle are congruent, $AB=PQ=\sqrt{8^2+4^2}=\mathbf{4\sqrt{5}}$.

7. We begin by seating Casey and Stacey. Casey has all eight seats to choose from, which leaves exactly six possibilities for Stacey, since she refuses to sit next to him, for a total of $8\cdot 6=48$ ways. Next we place Jenny and Lenny. If Jenny sits beside either Casey or Stacey then Lenny can take any of the remaining five seats, since the seat next to Jenny will already be occupied. But if Jenny sits in one of the four seats for which the adjacent seat is vacant, then Lenny will only be able to choose from among four of the remaining five seats. Hence these two may be seated in $2\cdot 5+4\cdot 4=26$ ways. Finally, the other four students can arrange themselves in any manner among the four remaining seats in $4!=24$ ways. The grand total is $48\cdot 26\cdot 24=\mathbf{29952}$ ways.

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⋆ Regional Level ⋆

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