



★ NATIONAL LEVEL ★

December 2009

# The Mandelbrot Competition

## Round Two Test

Name: \_\_\_\_\_

*Time Limit:*  
40 minutes

1. The tortoise and the hare are having another race. The hare hops ahead at 10 feet per second, while the tortoise advances only 1 foot every 4 seconds. The course is 500 feet long. The tortoise plods along steadily throughout the race, but the hare pauses for a half-hour nap partway through. By how many minutes does the victor win?		1										
2. Each of the quadratic functions below has one positive root. Which has the largest positive root? (Write A, B, C or D as your answer.) A. $x^2 + 4x - 8$ B. $x^2 + 4x - 9$ C. $x^2 + 5x - 8$ D. $x^2 + 5x - 9$		1										
3. Mel moves a marker along a row of squares, beginning in the square with the circle. On each turn he moves the marker 11, 18 or 25 spaces to either the right or left. What is the fewest number of moves needed to land in the square marked with an asterisk? ... <table><tr><td></td><td></td><td></td><td></td><td></td><td>○</td><td>*</td><td></td><td></td><td></td></tr></table> ...						○	*					2
					○	*						
4. A certain regular tetrahedron has three of its vertices at the points $(0, 1, 2)$ , $(4, 2, 1)$ and $(3, 1, 5)$ . Find the coordinates of the fourth vertex, given that they are also all integers.		2										
5. When $n = 1$ clearly $2^n - 2$ is divisible by 7, $4^n - 4$ is divisible by 9, $6^n - 6$ is divisible by 11, and $8^n - 8$ is divisible by 13. Find the next smallest positive integer $n$ for which this occurs.		2										
6. The positive real numbers $w$ , $x$ , $y$ and $z$ satisfy $w + 2x + 3y + 4z = 5$ . What is the minimum possible value of $w^2 + \frac{1}{2}x^2 + \frac{1}{3}y^2 + \frac{1}{4}z^2$ ?		3										
7. A cyclic quadrilateral has sides of length $3\sqrt{2}$ , $\sqrt{2}$ , 2 and 4 in that order around the quadrilateral. When opposite pairs of sides are extended they intersect at points $P$ and $Q$ . Compute length $PQ$ .		3										

SCORE: