



★ NATIONAL LEVEL ★

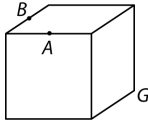
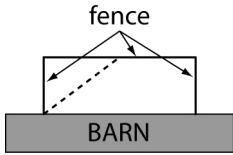
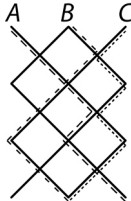
March 2011

The Mandelbrot Competition

Round Five Test

Name: _____

Time Limit:
40 minutes

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|---|---|---|
| 1. Imagine the plane passing through the points A , B and G shown at right. The intersection of this plane with the cube is a polygon. How many sides does this polygon have? |  | 1 |
| 2. At Mel's Mighty Market one can only buy fruit in bags. It is possible to purchase a bag with 3 apples and 5 bananas for \$4.00, or a bag with 7 apples and 9 bananas for \$7.50. Marge wishes to purchase at least 20 apples and at least 30 bananas. What is the least amount that she could spend? | | 2 |
| 3. Given a fraction $\frac{m}{n}$ between 0 and 1 written in lowest terms, create a new fraction $\frac{n-m+1}{n+m+2}$, then reduce it to lowest terms. Suppose repeating this process on the new fraction yields $\frac{m}{n}$ again. Determine both possible values of $\frac{m}{n}$. | | 2 |
| 4. A rectangular pen is built against the side of a barn. The total length of fencing used for the three sides is 20 meters. Suppose that the distance from one corner to the midpoint of the opposite side (indicated by the dashed line) is 9 meters. What is the area of the pen? |  | 2 |
| 5. Let α be the positive real solution to $x^3 + \frac{2}{5}x - 1 = 0$. Find the exact numerical value of $\alpha^2 + 2\alpha^5 + 3\alpha^8 + 4\alpha^{11} + \dots$. | | 3 |
| 6. Let $ABCD$ be a convex quadrilateral with $AB = 2$, $BC = 3$, $CD = 4$, $DA = 5$ and $area(ACD) - area(ABC) = 6$. Determine $\cos(\theta - \varphi)$, where $m\angle ABC = \theta$ and $m\angle CDA = \varphi$. | | 3 |
| 7. How many ways are there to draw three paths, starting at A , B and C , from the top to the bottom of the diagram following the solid lines? The paths must always head downward and may cross but may not overlap. The dotted paths illustrate one way. |  | 3 |

SCORE: