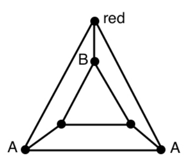
Answer Key		
1. Neeyanth	4. 63
2. 24	5. 66336
3. 20/7	6. 17/32
	7. 9261

1. At any given point in time, a person not holding the Frisbee has one throw for each catch (like Veer), unless they began with the Frisbee, in which case they have one more throw than catch (like Joshua). However, Neeyanth does not fall into either of these categories, so **Neeyanth** must be the one currently holding the Frisbee.

2. To begin, let's assume that the topmost dot is red. The other red dot must be at a vertex labeled A or B in order for the red dots to be joined by a segment. If it's at either dot labeled A, then there are two ways to finish the coloring from there, while if it's at dot B then there are four ways to finish the coloring, for a total of 8 valid colorings so far. The same logic applies if the topmost dot is blue or green, giving a grand total of **24** ways to color the dots.



3. Let the radius of the inner and outer circles be r and R. The shaded area is the difference in area of the two circles, so $\pi R^2 - \pi r^2 = 20\pi$. Meanwhile the total boundary is the sum of the two circumferences, meaning that $2\pi R + 2\pi r = 14\pi$. Dividing each equality by π gives

$$R^2 - r^2 = 20, \qquad 2R + 2r = 14.$$

Factoring and dividing by 2 leads to

$$(R+r)(R-r) = 20, \qquad R + r = 7.$$

The width of the ring is $R - r$, so we divide these equalities to obtain $R - r = \mathbf{20/7}$, or $\mathbf{2\frac{6}{7}}$.

4. We call Shivani's favorite integer x, and the classroom number c. According to Shivani, it is the case that

$$14(x+1) - c = x^2.$$

Stuart knows the room number c, but doesn't know the value of x, so he must solve the quadratic equation $x^2 - 14x + (c - 14)$. Ordinarily a quadratic would have two solutions, so Stuart would not have enough information to guess Shivani's integer. But there is an exception; namely, when there is a double root. This occurs only if the quadratic has the form $(x-7)^2 = 0$, or $x^2 - 14x + 49 = 0$. Hence we must have $c = \mathbf{63}$.

5. Clearly the final digit must be a 6, since any power of 2 will be even. Recalling our divisibility test for 4, (that the final two digits, viewed as a two-digit number, must be a multiple of 4) we can further deduce that our power of 2 must end 36, since 66 is not a multiple of 4. In fact, since our power of 2 is divisible by 8, the final three digits must be also, which implies that our number ends 336. Continuing in this manner, we realize that the final four and five digits must be multiples of 16 and 32, respectively, bringing us to 6336 and finally **66336**.

6. Suppose that there exists some person among the five who knows all the other phone numbers. This would mean that nobody else knows her number, so none of the others could reach her. In a similar manner, if some person does not know anyone else's phone number, then clearly this person can't reach any of the others. It turns out that these two scenarios are the only obstacles to the desired goal of having each person able to reach any other. (This is a nice exercise in graph theory, which we leave to the reader. It holds for any size group, not just five people.)

There are ten ways to choose two of the five people, and for each pair there are two possibilities for which person knows the other's phone number, for a total of $2^{10} = 1024$ arrangements. Our strategy is to count the number of "bad" arrangements, having one or both of the aforementioned obstacles. There are $5 \cdot 2^6$ ways to pick someone, have her know the other four phone numbers, then arrange the remaining six pairs in any fashion. In the same way, there are $5 \cdot 2^6$ ways for one person to know none of the other numbers. Finally, there are $20 \cdot 2^3$ ways to have both scenarios occur simultaneously, giving a total of

$$5(2^6) + 5(2^6) - 20(2^3) = 480$$

bad arrangments. Hence in the remaining $1024 - 480 = 544$ arrangements any person can reach any other person, so the desired probability is $544/1024 = \mathbf{17/32}$.

7. By assumption the cubic equation

$$x^3 - 2014x = x^2 + bx + c$$

has three real roots, call them u, v and w. Hence the points of intersection are

$$(u, u^2 + bu + c), \quad (v, v^2 + bv + c), \quad (w, w^2 + bw + c).$$

Using the formula giving the area of a triangle in terms of the coordinates of its vertices, we have

$$\begin{aligned} K &= \tfrac{1}{2}|u(v^2 + bv + c) + v(w^2 + bw + c) + w(u^2 + bu + c) \\ &\quad -w(v^2 + bv + c) - u(w^2 + bw + c) - v(u^2 + bu + c)|. \end{aligned}$$

Simplifying and factoring brings us to just

$$K = \tfrac{1}{2}|w - u| \cdot |v - u| \cdot |w - v|.$$

Without loss of generality say $u < v < w$. According to the statement of the problem we have $w - u = 42$ and $(v - u) + (w - v) = 42$. Since the sum of these latter two quantities is 42, their product is maximized when $v - u = 21$ and $w - v = 21$. Therefore the maximal area is

$$K = \tfrac{1}{2}(42)(21)(21) = 21^3 = \mathbf{9261}.$$

January 2014

⋆ National Level ⋆

The Mandelbrot Competition

Round Three Solutions

Greater Testing Concepts
PO Box 760
Potsdam, NY 13676

The Mandelbrot Competition
www.mandelbrot.org
info@mandelbrot.org