



★ NATIONAL LEVEL ★

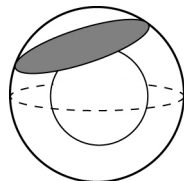
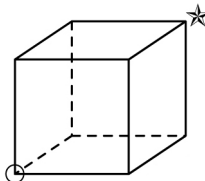
February 2011

# The Mandelbrot Competition

## Round Four Test

Name: \_\_\_\_\_

Time Limit:  
40 minutes

1. Faraz notices a bowl of cherries and eats half of them, plus three more. Next Meena discovers the bowl and does the same. Finally Yanson finds the bowl of cherries and also eats half of what is left, plus three more, at which point the bowl is empty. How many cherries were originally in the bowl?		①
2. It is possible to arrange five rectangles of sizes $1 \times 10$ , $2 \times 3$ , $4 \times 5$ , $6 \times 7$ , and $8 \times 9$ without overlap to completely fill out a single larger rectangle. What are the dimensions of the larger rectangle?		①
3. Find a quadratic having the form $x^2 + bx + c$ , where $b$ and $c$ are positive integers from 1 to 9, such that increasing $b$ by 1 yields a quadratic that can be factored, while increasing $c$ by 1 also yields a quadratic that can be factored.		②
4. Let $s$ and $t$ each represent a digit from 1 to 9 such that 1) If $s$ is odd then $t$ is spelled with four letters, 2) If $t$ is spelled with an 'f' then $s$ is even, and 3) If $s^2$ ends with 4 or 6 then $t^2$ ends with 3 or 7. Deduce the value of $t$ .		②
5. The large outer sphere in the diagram has a surface area of 2011, while the circular disc whose boundary lies on this sphere has area 76. A smaller concentric sphere is tangent to the circular disc. Find the surface area of the small sphere.		②
6. Consider the string of 14 letters $abab \cdots ab$ , with 7 $a$ 's and 7 $b$ 's. One is permitted to erase any four consecutive $a$ 's, erase any five consecutive $b$ 's, or replace any occurrence of $ab$ by $baa$ . What is the shortest possible string of letters that may be obtained? (Write the actual letters for your answer.)		③
7. Nathan places a marker at the circled corner of the given cube. On each turn he moves his marker along an edge of the cube to one of the three adjacent vertices, with each adjacent vertex being equally likely. On average, how many moves will it take him to reach the corner with the star?		③

SCORE: