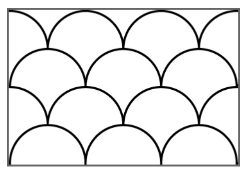
Answer Key

1. A
2. 6π
3. 2^{2008}
4. $2n+4$
5. 502
6. $7/16$
7. -3

1. The three quantities simplify to $4\sqrt{5}+\sqrt{5}=5\sqrt{5}$, $\frac{21}{4}\cdot\frac{8}{7}=6$ and $4-\pi-7+\pi=-3$. The first is not an integer, so the answer is **A**.

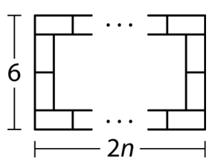
2. Based on the dimensions of the rectangle, we see that each circular arc in the diagram is part of a circle with diameter 1 cm. There are ten semicircles in the picture, which combine to create five whole circles. There are also four quarter-circles, which form one additional complete circle, for a total of six circles. Each circle has diameter 1cm, and hence circumference π cm, for a total length of $\boldsymbol{6\pi}$ cm.



3. Since $2^{10}=1024$ the numerator of the fraction equals 2^{1024}. On the other hand, the denominator becomes $(2^2)^{10}=2^{20}$. Subtracting exponents, the entire fraction simplifies to

$$\left(\frac{2^{1024}}{2^{20}}\right)^2=(2^{1004})^2=\mathbf{2^{2008}}.$$

4. One way to cover the rectangular track with dominoes is shown at left.

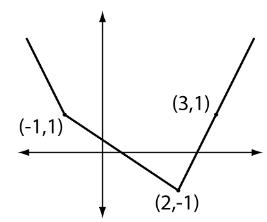


To begin, there are four vertical dominoes. The stretch of track along the top has length $2n$, hence can be covered by n dominoes. The same is true of the bottom portion of the track, for a total of $\boldsymbol{2n+4}$ dominoes.

5. We claim that each time Evan makes a cut he introduces four more edges to the overall edge count, regardless of where he chooses to make the cut. This occurs because he splits two of the existing edges in half, so where there used to be two edges there are now four, giving two additional edges. Furthermore, the cut itself creates two new edges, one on each of the resulting pieces. Hence there are four additional edges, as claimed. There are initially 3 edges, so after c cuts there will be $3+4c$ edges. We want $3+4c=2011$, which implies that $c=\mathbf{502}$.

6. Points A and B could be located along the top, bottom, left or right edge of the square, leading to a total of sixteen equally likely cases. In five of them A is definitely above B; for instance, when A is on the right edge and B is along the bottom edge. For another five cases B is always above A, and in two cases they are situated at the same height; namely, when A and B are both on the top or bottom edges. This leaves the four cases when A and B are both on a vertical edge of the square. By symmetry, A will be higher half the time, so in effect A has a larger y-coordinate in seven of the sixteen cases, giving a probability of $\mathbf{7/16}$.

7. The graph of $f(x)=|ax+b|+|cx+d|+e$ is reproduced below for reference. Since negatives within absolute value signs don't have an effect we may assume that a and c are positive. The first clue comes from the location of the corners on the graph. Recall that the graph of an absolute value function changes direction abruptly when the quantity within the absolute value reaches 0. Therefore our function must have the form



$$f(x)=|ax+a|+|cx-2c|+e$$

to produce corners at $x=-1$ and $x=2$. The second clue comes from the slopes of the graph. Near $x=0$ the quantity $ax+a$ is positive but $cx-2c$ is negative, so $f(x)=ax+a-cx+2c+e$ with slope $a-c$. Thus $a-c=-\frac{2}{3}$, since the graph has slope $-\frac{2}{3}$ there. However near $x=3$ both quantities are positive, so $f(x)=ax+a+cx-2c+e$ with slope $a+c$, therefore $a+c=2$. Solving gives $a=\frac{2}{3}$, $c=\frac{4}{3}$. Finally, we know that $f(2)=-1$, which enables us to compute $e=\mathbf{-3}$.

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⋆ Regional Level ⋆

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Greater Testing Concepts
PO Box 760
Potsdam, NY 13676

The Mandelbrot Competition
www.mandelbrot.org
info@mandelbrot.org