



★ NATIONAL LEVEL ★

February 2018

# The Mandelbrot Competition

## Round Five Test

Name: \_\_\_\_\_

Time Limit:  
40 minutes

1. Suppose the numbers $a, b, c, d$ are equal to 1, 2, 3 and 4 in some order such that (i) either $a > b$ or $a > c$ but not both, (ii) either $b > c$ or $b > d$ but not both, and (iii) $d > a$ . Compute $1000a + 100b + 10c + d$ .	1
2. Plot six points in the plane (no three collinear), then draw line segments connecting them so that each point is an endpoint of four segments. What is the least number of regions, including the outside, into which the plane may be divided by this process? (At right there are 19 regions.)	1
3. For a positive integer $n$ , let $f(n)$ be the least positive integer $b$ such that $b$ and $b + n$ are both composite. Thus $f(20) = 4$ since in the pairs 1, 21; 2, 22; 3, 23; 4, 24 the first instance of two composites is 4, 24. (Note that 1 is not a composite.) Find the sum of all <i>distinct</i> values of $f(n)$ .	2
4. Determine the value of $x$ for which the triangle with sides of length $x, x$ and 2.5 has the same area as the triangle with sides of length $x, x$ and $\sqrt{6}$ .	2
5. Find the remainder when the quantity below is divided by 61. $(1)(1 + 2)(1 + 2 + 3) \cdots (1 + 2 + 3 + \cdots + 59)$	2
6. Fran is moving a chip from the bottom left corner to the top right corner of a $4 \times 4$ grid. Given the allowed moves of length one or two units shown at right, in how many ways can she do this? A sample path is shown. (Note that two moves of length one unit is different than one move of length two units.)	3
7. Simplify the following expression, writing your answer in the form $a + bi$ . $\frac{(111 + i)(133 + i)(157 + i)(183 + i) \cdots (10101 + i)}{(122)(145)(170)(197) \cdots (10001)}$	3

SCORE: