



★ NATIONAL LEVEL ★

February 2014

The Mandelbrot Competition

Round Four Test

Name: _____

Time Limit:
40 minutes

1. Vincent owes Melissa \$14, but as he hands her a \$20 bill they drop it down a drain and it falls beyond their reach. Assuming they share the loss equally, how much does Vincent owe Melissa now?		①
2. Drake, James and Sarah are each thinking of a different two-digit number. Each person has a different tens digit, Drake's number is the smallest, and the product of their three numbers is less than 100,000. What is the largest possible value for Drake's number?		①
3. The vertices of a certain square are located on the lines $y = 1$, $y = 2$, $y = 3$ and $y = 4$, one vertex per line. What is the area of the square?		②
4. At a party there are seven individuals who each know precisely seven other people at the party, while the remaining guests each know exactly five other people. (If one person knows another, then that second person also knows the first.) What is the least number of people that could be at the party?		②
5. For positive integers $n \geq 2$, define $g(n)$ to be one more than the largest proper divisor of n . Hence $g(35) = 8$, since the proper divisors of 35 are 1, 5 and 7. For how many n in the range $2 \leq n \leq 100$ do we have $g(g(n)) = 2$?		②
6. In the game of Snozzle you begin with a marker at the left end of a row of fifteen squares with a star in the middle square. On each move you roll a fair six-sided die and move the marker that many squares in the direction of the star. (So you will either not reach the star, land on the star, or move past the star.) On average, how many rolls will it take to land on the star? <div style="display: flex; align-items: center; gap: 5px;"> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: black; border-radius: 50%;"></div> <div style="border: 1px solid black; width: 15px; height: 15px;"></div> <div style="border: 1px solid black; width: 15px; height: 15px;"></div> <div style="border: 1px solid black; width: 15px; height: 15px;"></div> <div style="border: 1px solid black; width: 15px; height: 15px;"></div> <div style="border: 1px solid black; width: 15px; height: 15px;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; text-align: center;">☆</div> <div style="border: 1px solid black; width: 15px; height: 15px;"></div> <div style="border: 1px solid black; width: 15px; height: 15px;"></div> <div style="border: 1px solid black; width: 15px; height: 15px;"></div> <div style="border: 1px solid black; width: 15px; height: 15px;"></div> <div style="border: 1px solid black; width: 15px; height: 15px;"></div> <div style="border: 1px solid black; width: 15px; height: 15px;"></div> </div>		③
7. Let φ be the acute angle with $\cos \varphi = \frac{4}{5}$, and let θ_1 , θ_2 and θ_3 be angles that sum to 360° . Suppose $\cos \varphi \cos \theta_1 \cos \theta_2 \cos \theta_3 - \sin \varphi \sin \theta_1 \sin \theta_2 \sin \theta_3 = \frac{2}{5}$. Determine the value of $\cos(\varphi + \theta_1) \cos(\varphi + \theta_2) \cos(\varphi + \theta_3)$.		③

SCORE: