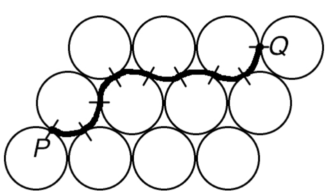
Answer Key	4. 0.999964
1. 18	5. $\tan\alpha = \cos\beta$
2. 5/7	6. 45 : 98
3. $8\pi/3$	7. 5

1. Since the two-digit numbers reading across have a greatest common divisor of 14, they must each be a multiple of 14, which limits their options to 14, 28, 42, 56, 70, 84, or 98. Likewise, each of the numbers reading down is a multiple of 3. This further restricts the possibilities to 14 and 28, 42 and 84, or 56 and 70. But if we use 14 and 28 then the numbers reading down are 12 and 48, which have a GCD of 12 rather than 3. The other order is no good either, since the GCD of 21 and 84 is 21. A similar issue arises with 42 and 84, but the pair 56 and 70 will work (in either order), giving a total digit sum of $5+6+7+0 = \mathbf{18}$.

2. Let x represent Maya's number. Since Simran's number is 1 less than twice Maya's number, Simran's number is $2x-1$. In the same way we find that Clara's number must be $2(2x-1)-1 = 4x-3$. But now Maya's number is 1 more than twice Clara's, therefore

$$x = 2(4x-3)+1 \quad\Longrightarrow\quad x = 8x-5 \quad\Longrightarrow\quad x = \mathbf{\frac{5}{7}}.$$

3. There are several possible paths of minimal length through the circles; one is depicted at left. However, all such paths involve exactly eight arcs extending a sixth of the way around a circle. These arcs are indicated by the short divider segments in the diagram. Since each arc has length $\frac{1}{6}(2\pi) = \frac{1}{3}\pi$, the total length of the path is $\mathbf{8\pi/3}$.



4. Squaring both sides of the given equation and using the fact that $(a+b)^2 = a^2+2ab+b^2$ leads to

$$(\sqrt{1-x})^2 + 2\sqrt{1-x}\sqrt{1+x} + (\sqrt{1+x})^2 = 2.012.$$

The left-hand side then simplifies to

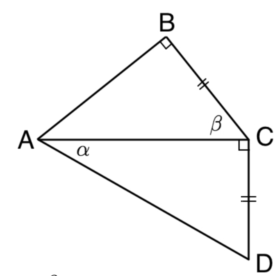
$$1 - x + 2\sqrt{(1-x)(1+x)} + 1 + x = 2.012.$$

Combining like terms and multiplying under the square root gives

$$2\sqrt{1-x^2} = 0.012 \quad\Longrightarrow\quad \sqrt{1-x^2} = 0.006.$$

Squaring again yields $1-x^2 = 0.000036$, and rearranging finally brings us to the answer of $x^2 = \mathbf{0.999964}$.

5. An accurate diagram is reproduced at right. The two right angles are marked, as are angles α and β. Furthermore, since $m\angle CBD = m\angle CDB$ we deduce that $\triangle CBD$ is isosceles, meaning that $CB = CD$, so these congruent sides are also marked in the diagram. It now becomes clear that the ratio CD/AC in $\triangle ACD$ will be the same as the ratio BC/AC in $\triangle ABC$, since $CB = CD$. The trigonometric functions corresponding to these ratios are $\tan\alpha$ and $\cos\beta$, therefore we may conclude that $\mathbf{\tan\alpha = \cos\beta}$.

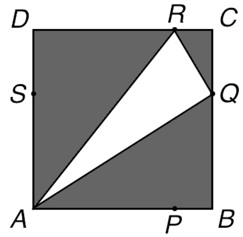


6. Suppose that we combine m cups of light red paint with n cups of pink paint. Since light red is $\frac{1}{5}$ white paint, while pink is $\frac{4}{7}$ white paint, we would have $\frac{1}{5}m + \frac{4}{7}n$ cups of white paint present in the mixture. Using the same reasoning, we would have $\frac{4}{5}m + \frac{3}{7}n$ cups of red paint. Therefore the ratio of white to red paint overall would be

$$\frac{\frac{1}{5}m + \frac{4}{7}n}{\frac{4}{5}m + \frac{3}{7}n} = \frac{7m+20n}{28m+15n}.$$

We want this expression to equal $\frac{5}{6}$. Cross-multiplying leads to the equality $42m+120n = 140m+75n$, or $45n = 98m$, so $\frac{m}{n} = \frac{45}{98}$. Therefore we should use a ratio $m\!:\!n$ of $\mathbf{45\!:\!98}$ of light red to pink paint.

7. The diagram is given at right for reference. We are not told any of the shorter lengths in the diagram, so we write $AP = DR = x$ and $AS = QB = y$. (These segments are congruent due to the fact that $\overline{PR} \parallel \overline{BC}$ and $\overline{QS} \parallel \overline{CD}$.) Since the square has side length 6, it follows that $RC = PB = 6 - x$ and $DS = CQ = 6 - y$. We next deduce that since $area(ARQ) = 13$ the remaining shaded area in the square has area $6^2 - 13 = 23$, hence



$$\tfrac{1}{2}(6)(x) + \tfrac{1}{2}(6)(y) + \tfrac{1}{2}(6-x)(6-y) = 23.$$

Upon multiplying out and combining like terms, we discover that

$$3x + 3y + 18 - 3x - 3y + \tfrac{1}{2}xy = 23,$$

which simplifies to $\frac{1}{2}xy = 5$. But $area(ASP) = \frac{1}{2}xy$, so we conclude that $area(ASP) = \mathbf{5}$.

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