



★ NATIONAL LEVEL ★

February 2012

The Mandelbrot Competition

Round Four Test

Name: _____

Time Limit:
40 minutes

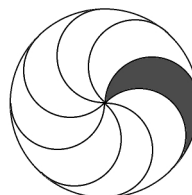
1. Brenna is thinking of a three-letter word. Seth guesses PAR, then ERA, and next PER. In each case Brenna indicates that exactly two of the three letters are contained in her word, and that at least one of them is in the correct position. What is Brenna's word?

1

2. A hat contains four cards, three of which are black on one side and red on the other, while the fourth is red on both sides. Nick chooses a card at random, looks at only one side, and observes that it is red. What is the probability that he has chosen the card that is red on both sides?

1

3. This pinwheel diagram consists of eight equally spaced semicircles, each with diameter 16 mm, within a larger circle of radius 16 mm, so that each semicircle has one endpoint at its center and the other endpoint on its circumference. What is the perimeter of the shaded region, in mm?

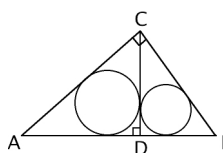


2

4. Simplify $(\log_2 3)(\log_6 7) + (\log_2 3) + (\log_6 7)$ to a single logarithm, writing your answer in the form $\log_b c$ for positive integers b and c .

2

5. In triangle ABC with right angle at C , let D be the foot of the perpendicular from C to side \overline{AB} . Suppose that the incircles of $\triangle ACD$ and $\triangle BCD$ have radii 5 and 3, respectively. What is the radius of the incircle of $\triangle ABC$?



2

6. In the game of Snatch two players begin with N pebbles in a bowl and alternate turns, removing either 1, 3 or 10 pebbles on each turn. The winner is the player to take the last pebble. For how many values of N in the range $1 \leq N \leq 2012$ can the first player guarantee a win by playing perfectly?

3

7. How many ordered pairs (a, b) satisfy $\left\lfloor \frac{a+100}{b} \right\rfloor = 1$ and $\left\lfloor \frac{b+100}{a} \right\rfloor = 2$, where a and b are positive integers and $\lfloor x \rfloor$ is the greatest integer function? (Thus $\lfloor x \rfloor$ is the largest integer not more than x , so $\lfloor \pi \rfloor = 3$ and $\lfloor 5 \rfloor = 5$, for example.)

3

SCORE: