



★ NATIONAL LEVEL ★

January 2019

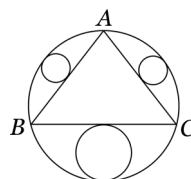
The Mandelbrot Competition

Round Four Test

Name: _____

Time Limit:
40 minutes

1. Compute the value $\frac{(3\frac{1}{2})(4\frac{1}{3})(5\frac{1}{4}) \cdots (9\frac{1}{8})}{(3\frac{1}{4})(4\frac{1}{5})(5\frac{1}{6}) \cdots (9\frac{1}{10})}$ as a fraction in lowest terms.	1
2. A square of side length 8 and a circle with radius 3 have the same center. Let region \mathcal{R} consist of the circle, the square, and the shaded area in between. Eight points are placed in \mathcal{R} such that the smallest distance between two points is $\sqrt{17}$. What is the second smallest distance between any two points?	1
3. A box contains two copies of each letter of the alphabet, on 52 cards total. Seven letters are removed, which form the word 'FRIENDS'. Exactly one of these letters is thrown away, then another is taken out of the box to replace it. How many (unordered) sets of letters could possibly result afterwards?	2
4. Find the triple of real numbers x, y, z , none of which equal 0, that make exactly three of these four equations true. $\begin{array}{rcl} 2y & = & x + 4 \\ 3x + 2z + 12 & = & 6y \end{array} \qquad \begin{array}{rcl} 5y + 5 & = & 2x + 2z \\ z + 6 & = & 3y \end{array}$	2
5. The ellipse $\frac{x^2}{100} + \frac{y^2}{19} = 1$ is printed on a 20×20 sheet of paper so that the ellipse is centered and its axes are parallel to the edges of the paper. The sheet of paper is then rolled up into a cylinder so that the two foci of the ellipse are physically touching one other. What is the volume of the cylinder?	2
6. How many polynomials $P(x)$ are there (including degree zero) which have nonnegative integer coefficients and give $P(P(1)) = 17$?	3
7. Inscribe isosceles $\triangle ABC$ in a circle. Then construct circles tangent to the midpoints of \overline{AB} , \overline{AC} and \overline{BC} , and internally tangent to the first circle. Given that the radii of the three smaller circles are 5, 5, and 9, what is the area of $\triangle ABC$?	3



SCORE: