November 2010

★ National Level ★

The Mandelbrot Competition

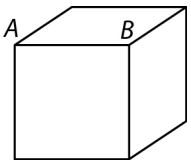
Round One Test

Time Limit: 40 minutes

Name: _____

1. Find the value of x satisfying $(x-7)(17-10) = (x-10)(17-7)$. (1)

2. A *quarter-turn* of a cube consists of rotating the cube by $90°$ in either direction around any line through the centers of opposite faces. What is the minimum number of quarter-turns necessary to swap the two corners labeled A and B?

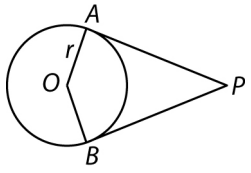


(1)

3. We say that a positive integer is *nimble* if it can be written as a perfect cube minus a prime. For instance, 12 is nimble since $12 = 5^3 - 113$. What is the next nimble positive integer after 12? (2)

4. Abby sells widgets for \$30 apiece. She plans to discontinue the current model and instead offer two new types of widgets, one of which will cost \$8 more than the other. She does not expect a change in the total number of sales, but anticipates that 75% of the units sold in the future will be the cheaper model. If she wants revenue to increase by 10%, how much should she charge for the less expensive type of widget? (2)

5. Two points A and B are located on a circle with center O and radius r. The tangents to the circle at A and B intersect at a point P. If $AB = 12$, $OP = 13$, and $r < 10$, then compute r.

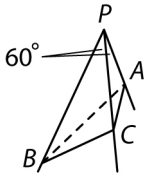


(2)

6. Which of the numbers listed below is the largest? (3)

$$50\sqrt{1},\ 49\sqrt{2},\ 48\sqrt{3},\ \ldots,\ 2\sqrt{49},\ \sqrt{50}$$

7. Three rays in space create three $60°$ angles at their vertex P. Points A, B and C are selected along the rays so that lengths $PA < PC < PB$ are positive integers and so $m\angle ACB = 90°$. If $PB = 2010$ and PC is odd, then determine PA.



(3)

SCORE: