November 2013

⋆ National Level ⋆

The Mandelbrot Competition

Round One Test

Time Limit: 40 minutes

Name: _____

1. Claire writes down all the positive integers from 10 to 99, inclusive. She next adds together all the numbers ending with a 3, 1, 4, 5 or 9 and crosses them off her list. She then adds together all the remaining numbers. How much larger is the second total than the first? (1)

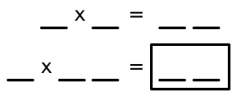
2. For a given $b > 0$, draw the lines $y = \frac{1}{2}x + b$, $y = \frac{1}{2}x + b + 5$ and $x = 6$. Together with the x and y axes, these lines form two non-overlapping regions in the first quadrant. For which value of b do the regions have equal area? (1)

3. Find the largest natural number n below 50 such that

$$\text{LCM}(n, n+1, \ldots, 50) = \text{LCM}(1, 2, 3, \ldots, 50),$$

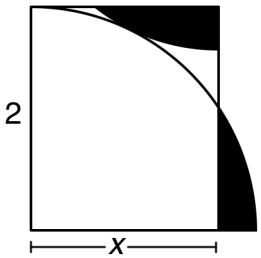
where LCM stands for least common multiple. (2)

4. It is possible to place all the digits from 1 to 9 into the blanks (using each digit once) so that both multiplication statements are correct. What two-digit number appears within the rectangle? (2)



5. Let α, β and γ be the three roots of the polynomial $x^3 + 20x^2 - 13x - 14$, and let $p(x)$ be the monic cubic polynomial whose roots are $\alpha + \beta$, $\alpha + \gamma$ and $\beta + \gamma$. Compute $p(0)$. (2)

6. A quarter of a circle with radius 2 is partially enclosed in a 2 by x rectangle. The portion of the circle outside of the rectangle (colored black) can be flipped over and moved to exactly fit in the remaining space within the rectangle, tangent to the circle, as shown at right. For what value of x does this occur? (3)



7. Let $0 < x < 1$ and $2 < y < 3$ be chosen independently and uniformly at random. Find the probability that $\left\lfloor \log_3\left(\frac{y}{x}\right) \right\rfloor$ is odd. (3)

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SCORE: _____