February 2018

⋆ Regional Level ⋆

The Mandelbrot ℂompetition

Round Five Test

Time Limit: 40 minutes

Name: _____

1. Calculate the value of the following expression.

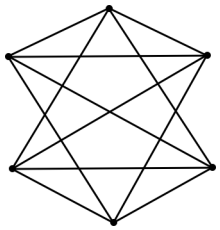
$$(1) - (1+2) + (1+2+3) - \cdots + (1+2+3+\cdots+19)$$

(1)

2. Suppose the numbers a, b, c, d are equal to 1, 2, 3 and 4 in some order such that (*i*) either $a > b$ or $a > c$ but not both, (*ii*) either $b > c$ or $b > d$ but not both, and (*iii*) $d > a$. Compute $1000a + 100b + 10c + d$.

(1)

3. Plot six points in the plane (no three collinear), then draw line segments connecting them so that each point is an endpoint of four segments. What is the least number of regions, including the outside, into which the plane may be divided by this process? (At right there are 19 regions.)



(2)

4. For a positive integer n, let $f(n)$ be the least positive integer b such that b and $b+n$ are both composite. Thus $f(20) = 4$ since in the pairs 1, 21; 2, 22; 3, 23; 4, 24 the first instance of two composites is 4, 24. (Note that 1 is not a composite.) Find the sum of all *distinct* values of $f(n)$.

(2)

5. Determine the value of x for which the triangle with sides of length x, x and 2.5 has the same area as the triangle with sides of length x, x and $\sqrt{6}$.

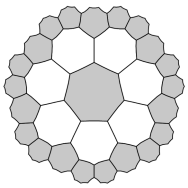
(2)

6. Determine the remainder when the quantity below is divided by 61.

$$(1)(1+2)(1+2+3)\cdots(1+2+3+\cdots+59)$$

(3)

7. The diagram shows a portion of a tiling by heptagons, with three heptagons meeting at every vertex. The first ring around the center has 7 heptagons, while the second ring has 21. How many heptagons would the fourth ring have?



(3)

SCORE: _____