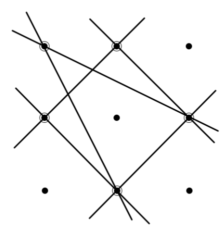
Answer Key	4. 3/2
1. 47	5. 1/4
2. 6	6. 99
3. 3891676	7. 25/4

1. It may be tempting to solve the equation; however, its solutions contain radicals that are hard to work with. Here is a cleaner way. First expand $(x+3)(x+4)$ to get $x^2+7x+12$. We are given $x^2+7x=35$, so adding 12 to both sides gives $x^2+7x+12 = \mathbf{47}$, which is our answer.

2. We choose to minimize the number of horizontal and vertical lines, which are easier to account for, since that will maximize the number of non-horizontal and non-vertical lines. In the rows, there could be 1 dot in one of the rows and 2 dots in the others, which creates 2 horizontal lines, or 3 dots in one of the rows and 1 dot in the others, which creates 3 horizontal lines (that all look the same). So at least 2 horizontal lines are created. Likewise, at least 2 vertical lines are created, so the number of horizontal and vertical lines is at least $2+2=4$. The total number of lines created by the points is 10, so the maximum number of lines that can be drawn is $10-4=\mathbf{6}$, as illustrated at right.



3. Note that the numbers 3, 6, 6, 9 are all divisible by 3, so they must occupy the first, third, fifth and seventh squares. Also, 8 must not be adjacent to either of the 6s, which restricts placement of the 6s to one side or the other. To minimize our number, we try placing the smallest possible numbers from left to right. The smallest number that can be placed in the first square is 3. This rules out the 6s being on the left side, so they must be on the right, and thus 8 is the second digit. The rest of the squares can easily be deduced, with the 1 preceding the 7, and the resulting number is **3891676**.

4. Using addition of logarithms, we obtain

$$(\log_{28} 216)(\log_6 2 + \log_6 \sqrt{7}) = (\log_{28} 216)(\log_6 2\sqrt{7})$$

Note that $2\sqrt{7} = \sqrt{28} = 28^{1/2}$, so $\log_6 2\sqrt{7} = \log_6 28^{1/2} = \frac{1}{2}\log_6 28$. Finally, using the logarithm change-of-base formula, we get

$$(\log_{28} 216)(\tfrac{1}{2}\log_6 28) = \tfrac{1}{2}(\log_6 28)(\log_{28} 216) = \tfrac{1}{2}\log_6 216 = \mathbf{\frac{3}{2}}.$$

5. This problem seems difficult to tackle directly, so consider a simpler problem: given a *diameter*, what is the probability that our line segment crosses it? A diameter divides the circle into congruent regions, so a random point is equally likely to be in either half. The probability that two random points are in different halves (producing a segment that crosses the diameter) is $\frac{1}{2}$, as the second random point is as likely as not to be in the same half as the first random point.

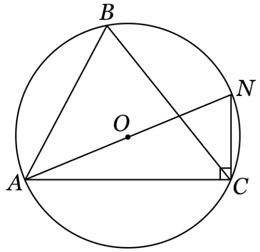
Now consider a radius, which is half the diameter. By symmetry, each case of a line segment intersecting the radius corresponds to another case in which the line segment does not cross the radius, by reflecting the diagram from top to bottom. (The segment might pass through the center of the circle, but this happens with probability zero.) Hence if a line segment passes through the diameter, there is a $\frac{1}{2}$ chance that it passes through the radius, giving an answer of $(\frac{1}{2})(\frac{1}{2}) = \mathbf{\frac{1}{4}}$.

6. Let the number of books and bookmarks Lillie receives be a and b, respectively; thus the number of ways to choose a book and bookmark is ab. The number of books and bookmarks Selene has left is $25-a$ and $16-b$, giving $(25-a)(16-b)$ ways for her to choose a book and bookmark. The problem can thus be written algebraically as

$$ab = (25-a)(16-b)+1.$$

Expanding and simplifying gives $16a+25b=401$, at which point guess-and-check by varying b is an efficient way to finish the problem. Trying out values of b up to 16 yields the only positive integer set of solutions $a=11$, $b=9$. Therefore the number of ways that Lillie can choose a book and bookmark is $11 \cdot 9 = \mathbf{99}$.

7. In the diagram below we have drawn diameter $\overline{AN}$ and segment $\overline{CN}$. Since an angle inscribed in a semicircle always measures $90°$, we also label $\angle ACN$ as a right angle. This immediately suggests a solution strategy; if we could determine length AN, then we could find CN via the Pythagorean Theorem.



A standard method to find the diameter of a circumcircle is to combine Hero's area formula with the circumradius area formula, as follows. The semiperimeter is $s = \frac{1}{2}(13 + 14 + 15) = 21$, hence

$$K = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{(21)(8)(7)(6)} = 84,$$

where K represents *area*(ABC). But we also know that

$$K = \frac{abc}{4R} \qquad \Longleftrightarrow \qquad R = \frac{13 \cdot 14 \cdot 15}{4 \cdot 84} = \frac{65}{8}.$$

We deduce that the diameter length $AN = 2R = \frac{65}{4}$. We could now finish via Pythagorean, but it's quicker to observe that $\triangle ACN$ is simply a 5–12–13 triangle scaled by a factor of $\frac{5}{4}$ up to a $\frac{25}{4}$–15–$\frac{65}{4}$ right triangle. Hence the answer is $CN = \mathbf{\frac{25}{4}}$.

Problem Credits

		5. Dr.V
1. Dr.V	3. Josh P.	6. Isaac L
2. Jon S.	4. Josh P.	7. Jon S.

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⋆ Regional Level ⋆

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