January 2011

⋆ National Level ⋆

The Mandelbrot Competition

Round Three Test

Time Limit: 40 minutes

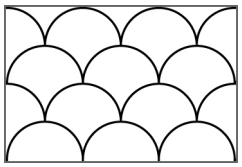
Name: _____

1. Which of the following is not an integer? (Write A, B or C as your answer.)

   A. $\sqrt{20}+\sqrt{5}$ B. $(5+\frac{1}{4})(1+\frac{1}{7})$ C. $(4-\pi)-(7-\pi)$

(1)

2. The rectangle here measures 3 cm by 2 cm. Compute the total length, in cm, of all the circular arcs inside the rectangle. (Don't include the perimeter of the rectangle.)



(1)

3. Simplify $\left(\dfrac{2^{(2^{10})}}{(2^2)^{10}}\right)^2$ to the form 2^n, where n is a positive integer.

(2)

4. Evan has a large triangular sheet of paper. He cuts from the midpoint of one edge straight to the midpoint of another edge. He then selects one of the resulting pieces and again cuts from the midpoint of one edge to the midpoint of another edge. He repeats this process until there are a total of 2011 edges among all the pieces. How many cuts did he make?

(2)

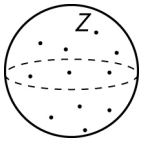
5. Consider the unit square with vertices at $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$. Suppsose that we choose two points A and B independently and randomly around the perimeter of this square. What is the probability that point A is higher than point B; i.e. has a larger y-coordinate?

(2)

6. Let $f(x)=x^4-49x^2-14x-1$ and let $g(x)=ax+b$. Find positive integers a and b for which $f(g(x))$ is divisible by $x^2+9x+19$.

(3)

7. Let P_1, P_2, …, P_{10} and Z be eleven points located on the surface of a sphere so that no four points lie on the same circle. A plane passing through Z, but none of the other points, splits the points P_1 to P_{10} into two subsets, one of which might be empty. How many partitions of the points into two subsets can be achieved in this manner?



(3)

SCORE: