Answer Key	
1. B	4. 74°
2. 8	5. 81
3. 2125	6. $3, 5$
	7. 4

1. To compare the lines we write each equation in $y = mx + b$ form. The given equation becomes $y = -\frac{2}{3}x + \frac{4}{3}$. Hence the slope of a perpendicular line is the negative reciprocal of $-\frac{2}{3}$, or $\frac{3}{2}$. Since the middle equation may be rewritten as $y = \frac{3}{2}x - \frac{5}{4}$, choice **B** is correct.

2. Suppose the second statement were true. Since the only pair of digits that multiply to 28 are 4 and 7, these numbers would have to appear on the reverse sides of the cards. But their sum is 11, meaning both statements would be true, contradicting the fact that there is a 4 on one of the cards, making that card's statement false.

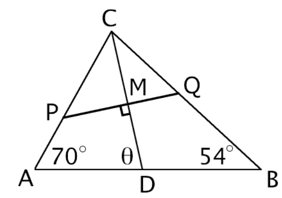
Therefore the second statement must be false, so a 4 appears on that card. By the same reasoning, if the first statement were true then we would once again have a 4 and a 7, leading to the same contradiction as before. Hence both statements must be false, implying that both cards feature a 4, so the desired sum is **8**.

3. We need only determine how the new total area compares to the old. Suppose that each outlet originally had radius R, making the combined area equal to $2\pi R^2$. The new radii would be $2R$ and $\frac{1}{2}R$ after changing the sizes of the outlets, for a combined area of

$$\pi(2R)^2 + \pi(\tfrac{1}{2}R)^2 = 4.25\pi R^2.$$

Since the area has increased by a factor of 2.125 so will the flow rate, giving $2.125(1000) = \mathbf{2125}$ gpm.

4. Imagine unfolding the paper again after folding point C on top of point D. By the nature of folds, the crease $\overline{XY}$ will be perpendicular to $\overline{CD}$. (The crease will bisect $\overline{CD}$ also, but we won't need this fact.)



Since there are a total of 360° in quadrilateral $APMD$ we know that $m\angle APM = 200^\circ - \theta$. But if quadrilateral $APQB$ is to be cyclic, then the sum of its opposite angles must be 180°. Hence $(200^\circ - \theta) + 54^\circ = 180^\circ$, which leads to $\theta = \mathbf{74^\circ}$.

5. We first combine each pair of numbers having the same tens digit, like (53)(57). One quickly discovers that each such product ends with the digits 21. This occurs because

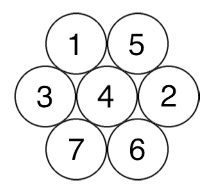
$$(10n + 3)(10n + 7) = 100n^2 + 70n + 30n + 21 = 100(n^2 + n) + 21.$$

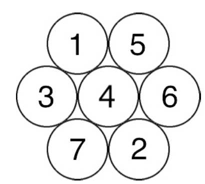
Therefore to determine the final two digits of the entire product we must find the last two digits of 21^9. It is relatively easy to find a pattern among the successive powers 21, 21^2, 21^3, etc. Alternately, one may write

$$(1 + 20)^9 = 1 + 9(20) + 36(20)^2 + \cdots + 20^9$$

using the Binomial Theorem. Only the first two terms contribute to the units and tens digits, and their sum is 181, so the answer is **81**.

6. It is surprisingly tricky to stumble upon even a single configuration of the seven digits satisfying the given condition. A careful search reveals a total of two possible solutions, not counting rotations and reflections. These are shown below.

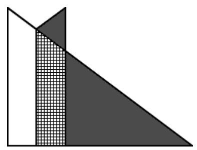




Note that in either case the same two digits flank the 1, so our answer is **3,5**. (It is also fine to list the digits in the other order, as in **5,3**.)

7. Since the total area of the original triangular region is fixed, minimizing the area of the resulting shape is equivalent to maximizing the

region of overlap, indicated by the cross-hatch pattern in the figure at left. Let us superimpose a coordinate system so that the vertices of the



original triangle have coordinates $(0,0)$, $(4,0)$ and $(0,3)$. It follows that the hypotenuse has equation $-\frac{3}{4}x+3$. Suppose the vertical fold is made at a certain x-value. Then the right edge of the overlap region is located at $2x$. We deduce that this region is a trapezoid with heights $-\frac{3}{4}x+3$ and $-\frac{3}{4}(2x)+3$, and width x. Therefore its area is

$$\frac{x}{2}\left((-\tfrac{3}{4}x+3)+(-\tfrac{3}{4}(2x)+3)\right)=-\frac{9}{8}x^2+3x.$$

The graph of this equation is a downward opening parabola with vertex at $(\frac{4}{3},2)$. In otherwords, the maximum area for the region of overlap is 2, occuring when we fold at $x=\frac{4}{3}$. Since the total area of the original triangle is 6, we conclude that the minimal folded area is $6-2=\mathbf{4}$.

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⋆ Regional Level ⋆

The Mandelbrot Competition

Round Three Solutions

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