



★ NATIONAL LEVEL ★

November 2018

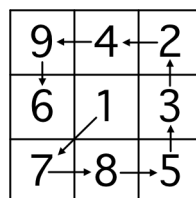
The Mandelbrot Competition

Round Two Test

Name: _____

Time Limit:
40 minutes

1. Using the given numbers, start a path at 1, move to one of its eight neighbors, then circle around in either direction until every number is visited once. The *weight* is the total of the positive differences between consecutive numbers in the path. Thus the path shown has weight 23. What is the minimum possible weight of a path with these numbers?



1

2. Rose and Karl walk along a 7 mile trail. Rose walks at 3 miles per hour, and Karl walks 2 miles per hour. Since Karl falls behind, Rose waits for him at each trail marker. The trail markers are located 1.7, 3.6, 5.5, and 7 miles into the trail. For how many minutes in total does Rose wait for Karl?

1

3. Suppose that α and β are measures of acute angles, with $0^\circ < \alpha < \beta < 90^\circ$. If $\sin \alpha = \cos \beta$ and $\tan(2\alpha) = \tan(3\beta)$, then find α , in degrees.

2

4. Find the smallest positive integer n such that n is a multiple of 2018, and the number of positive divisors of n (including both 1 and n) is exactly 2018. You may write your answer in exponential, i.e. factored, form.

2

5. Suppose there is a $\frac{1}{3}$ probability that a person knows that cashews in fact come from a fruit. Given ten people at a party, how many pairs of people, on average, will there be who both know the truth concerning the cashew?

2

6. Evaluate the following sum, writing your answer as a reduced fraction.

$$\frac{100 + 101}{100^2 \cdot 101^2} + \frac{101 + 102}{101^2 \cdot 102^2} + \frac{102 + 103}{102^2 \cdot 103^2} + \cdots + \frac{199 + 200}{199^2 \cdot 200^2}$$

3

7. Quadrilateral $ABCD$ has side lengths $AB = 6$, $BC = 7$, $CD = 5$, and $DA = 4$. Point P is situated in $ABCD$ such that $m\angle APB = m\angle CPD = 90^\circ$ and $m\angle BPC = 2m\angle APD$. What is the positive difference between the areas of triangles APD and BPC ?

3

SCORE: