



★ NATIONAL LEVEL ★

December 2018

# The Mandelbrot Competition

## Round Three Test

Name: \_\_\_\_\_

Time Limit:  
40 minutes

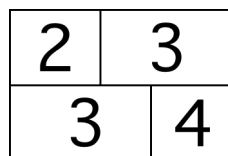
1. Two of the digits from 0 to 9 are used to create a two-digit number that is equal to the sum of the other eight unused digits. Find this two-digit number.

1

2. Find all positive real numbers  $x$  such that  $5^x + 3 < 2^{2x+1}$ , expressing your answer as an inequality.

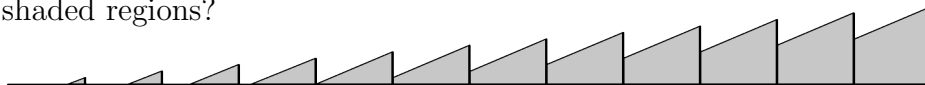
1

3. How many ways are there to place a 1, 2, 3, 4 or 5 in each rectangle at right so that adjacent rectangles (whose perimeters overlap) have digits differing by at most 1? It is fine to use digits more than once, as shown.



2

4. In the diagram below the vertical segments are spaced 1 unit apart, are perpendicular to the base, and have heights of  $\frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \dots, \frac{12}{12}$  from left to right. If each slanted segment has slope  $\frac{5}{12}$ , then what is the total area of all the shaded regions?



2

5. A lattice point is a point in the plane having integer coordinates. Suppose that a certain quadrilateral has all four vertices at lattice points, encloses eight lattice points in its interior, and has sides of length 11,  $2\sqrt{10}$ ,  $3\sqrt{13}$  and  $4\sqrt{2}$  in some order. Determine its area.

2

6. Let  $\mathcal{S}$  be the set of rectangles that can be formed using one or more squares from a  $9 \times 9$  grid of unit squares. Let  $R_1$  and  $R_2$  (not necessarily distinct) be chosen randomly from  $\mathcal{S}$ , and let  $A$  be the area inside the union of  $R_1$  and  $R_2$ , but outside their intersection. What is the probability that  $A$  is odd?

3

7. Starting with  $2^a 3^b$ , for positive integers  $a$  and  $b$ , repeatedly multiply by the first fraction in the list  $\frac{11}{13}, \frac{26}{77}, \frac{1}{11}, \frac{35}{2}, \frac{11}{3}, \frac{1}{7}$  for which the new number is an integer. Thus starting with 36, we find that  $36 \cdot \frac{11}{13}$ ,  $36 \cdot \frac{26}{77}$ , and  $36 \cdot \frac{1}{11}$  are not integers, so the next number is  $36 \cdot \frac{35}{2} = 630$ . When this is no longer possible, the resulting number has the form  $5^k$ . Find  $k$  in terms of  $a$  and  $b$ .

3

SCORE: