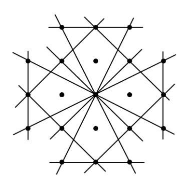
Answer Key	
1. 9	4. $7\pi/3$
2. 14	5. 236
3. 273	6. 71
	7. $3/4$

1. The least common denominator is 35, so the sum is

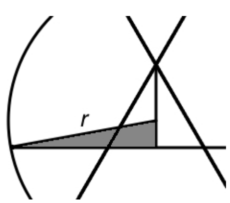
$$\frac{a}{7}+\frac{b}{35}=\frac{5a}{35}+\frac{b}{35}=\frac{5a+b}{35}.$$

If we can arrange for $5a+b=14$ then the sum would reduce to $2/5$, which has a 2 in the numerator, as desired. Since a and b are odd positive integers, we choose $a=1$ and $b=\mathbf{9}$. (Using the fact that b is not a multiple of 5, it is possible to show that this is the only solution.)

2. The illustration to the right shows all **14** lines that pass through exactly three points in the grid.



3. One quickly realizes that no number from 250 to 259 can work, since twice such a number begins with a 5, repeating that digit. Even numbers are also not likely to work, since they produce a lot of even digits. Checking the other possibilities leads one to discover before long that the first three multiples of **273** are 273, 546, 819, which contain each of the digits from 1 to 9 exactly once. (Note that 192, 384, 576 is another such triple.)



4. Perhaps the most direct approach is to use the Pythagorean Theorem to compute the radius of the circle. Focusing on the part of the diagram shown, the altitude of the middle equilateral triangle is $\sqrt{3}/2$ using 30°–60°–90° triangle facts. The center of the triangle (and hence of the entire circle) is one-third of the way up this altitude, which is a distance of $\sqrt{3}/6$. Furthermore, half the horizontal chord has length $3/2$. This gives the two legs of the shaded right triangle, so the hypotenuse is

$$r=\sqrt{\left(\frac{\sqrt{3}}{6}\right)^2+\left(\frac{3}{2}\right)^2}=\sqrt{\frac{1}{12}+\frac{9}{4}}=\sqrt{\frac{7}{3}}.$$

Hence the area of the circle is $\pi r^2=\mathbf{7\pi/3}$.

5. Since our positive integer requires four digits in base five it must be at least 125, since $125=1000_5$. But it has only seven binary digits, so it can be at most 127, since $127=1111111_2$. Together this limits the possibilities to either 125, 126 or 127. But we have just seen that 127 is a palindrome in base two, while $126=1001_5$ which is also a palindrome. Hence the answer is 125, which is written as **236** in base seven.

6. The table below shows the various combinations of \$1, \$2 and \$5 bills that total to \$8.

\$5	1	1	0	0	0	0	0
\$2	1	0	4	3	2	1	0
\$1	1	3	0	2	4	6	8

Consider the second column, for sake of illustration. Since each bill comes in two colors, there are four ways to have three \$1 bills: all blue, two blue and a green, one blue and two green, or all green. There are also two ways to have a \$5 bill: either blue or green, giving a total of $(4)(2)=8$ possibilities. This amounts to adding 1 to each entry of the second column and multiplying the results. The same approach works for the remaining columns, giving an overall total of

$$8+8+5+12+15+14+9=\mathbf{71}.$$

7. To solve for x we multiply the first equation by $\log_5 21$ and the second equation by $\log_5 13$, then subtract. The y terms now cancel, leaving

$$(\log_5 21)(\log_{21} 48)x-(\log_5 13)(\log_{13} 3)x=$$
$$(\log_5 21)(\log_{21} 56)-(\log_5 13)(\log_{13} 7).$$

Using the fact that $(\log_a b)(\log_b c) = \log_a c$, the above equation becomes

$$(\log_5 48 - \log_5 3)x = \log_5 56 - \log_5 7.$$

Now we employ the subtraction law $\log_a b - \log_a c = \log_a(\frac{b}{c})$ to obtain

$$(\log_5 16)x = \log_5 8 \quad \Longrightarrow \quad x = \frac{\log_5 8}{\log_5 16} = \frac{3\log_5 2}{4\log_5 2} = \frac{\mathbf{3}}{\mathbf{4}}.$$

(Note that answers involving $\log_5$ are not in simplest form, so should not receive credit.)

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⋆ Regional Level ⋆

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