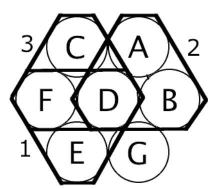
Answer Key			
1.	288	4.	$300\sqrt{3}$
2.	3	5.	2.93
3.	\$12	6.	32/45
		7.	$\sqrt{14}/7$*

*See below for an alternate acceptable answer.

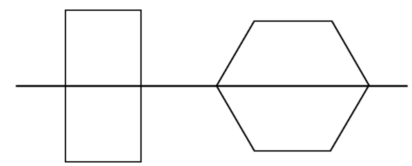
1. For a positive integer to be divisible by 9 the sum of its digits must also be a multiple of 9. But if all the digits are even then the sum of the digits will be also, making this sum at least 18. Since each digit is at most 8 we will need at least three digits; the first time the digits add to 18 is for **288**.

2. Note that any move changes the position of two of the outer six letters. Since five of those letters will need to be moved (all but the G) we require at least three moves. In fact, the puzzle can be solved in three moves as shown at right: rotate each of the highlighted triangular blocks in a counterclockwise direction in the order indicated. Hence the minimum number of moves is **3**.



3. According to the statement of the problem, Zach's earnings would increase by \$260 if he were to sell twenty more whizdoodles. Therefore Zach must currently be selling whizdoodles for \$13. Since he raised his price by \$1, the original cost must have been **\$12**.

4. Contrary to intuition, a square does *not* give the region of maximal area in this sort of situation. To understand why, imagine drawing a mirror image of the enclosed region on the other side of the barn wall. This will simply double the area of the chicken coop, so an equivalent problem would be to enclose as large an area as possible with six straight lengths of fencing. It makes sense to come as close to a circle as possible, which is achieved by a regular hexagon. Therefore the answer to the original problem is "half a hexagon." Its area is equal to that of three equilateral triangles of side length 20, or $3(20^2\sqrt{3}/4) = \mathbf{300\sqrt{3}} \approx 519.6$, considerably larger than the 400 we would obtain with a square.



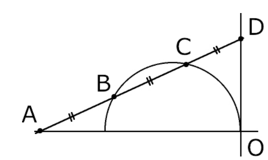
5. We wish to approximate the solution to $3^x + 2x = 31$. In order to match the form of the equation whose solution we are given, we multiply both sides by 3, then add 6, yielding

$$3^x + 2x = 31 \quad\Longrightarrow\quad 3^{x+1} + 6x = 93 \quad\Longrightarrow\quad 3^{x+1} + 6(x+1) = 99.$$

In order to satisfy this latter equation we are told to take $x + 1 \approx 3.93$. Therefore we want $x = \mathbf{2.93}$.

6. Gina's worst strategy is to choose numbers such as 28 and 93, which rule out two units and two tens digits. In order to do better she should try a pair such as 28 and 38 or 93 and 94, which leave more possibilities for winning when the random number is chosen. In the former case the probability of not matching is $\frac{7}{9} \cdot \frac{9}{10}$, while in the latter case the probability is $\frac{8}{9} \cdot \frac{8}{10}$. The latter value is slightly higher, for an optimal probability of $\frac{8}{9} \cdot \frac{8}{10} = \mathbf{\frac{32}{45}}$.

7. Scale the figure so that the length of each congruent segment is 1. Note that this transformation does not change the slope of the line, so it is a valid simplification of the problem. For convenience, we label the various points in the diagram as shown. Now apply the Power of a Point Theorem to point D, which states that $(DC)(DB) = (DO)^2$. Since $DC = 1$ and $DB = 2$ we find that $DO = \sqrt{2}$. Next the Pythagorean Theorem implies that $(AO)^2 + (DO)^2 = (AD)^2$, hence $(AO)^2 = 3^2 - (\sqrt{2})^2 = 7$, so $AO = \sqrt{7}$. We are now in a position to determine the slope, which is equal to $\sqrt{2}/\sqrt{7} = \mathbf{\sqrt{14}/7}$ (or $\sqrt{\mathbf{2/7}}$).



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