Team Play Topics

ROUND ONE

The first section of the Round One Mandelbrot Team Play is reproduced below. A list of topics and practice problems are also provided to aid in preparation. Note that these problems are not meant to serve as a precise indicator of the problems that will appear on the contest. However, students who understand how to solve them should be able to make significantly more progress than they might have otherwise. So work hard on the problems, and good luck on the Team Play!

Facts: The *minimum function* min returns the smallest of two or more real numbers, giving the common least value in case there is a tie. Thus $\min(\pi, \sqrt{10}) = \pi$, $\min(-7, -8, -9) = -9$, and $\min(14, 20, 14) = 14$. The graph of $\min(2x, 6)$ is a *continuous, piecewise linear function*, meaning that in the vicinity of each x-value the graph is a line, except at *nonlinear points*, where two lines with different slopes meet. Thus $\min(2x, 6)$ has a nonlinear point at $x = 3$.

TOPICS: Minimum function, piecewise linear functions, graphing linear functions of two variables

Practice Problems

1. Draw the graph of $\min(2x, x+3, 10)$ and identify all nonlinear points.

2. For what value of x does $\min(2x, x+3, 10)$ equal 5? Where does it equal 9? What about 14?

3. Find a formula for the piecewise linear function whose graph passes through both the origin and the point $(30, 18)$, and which has nonlinear points at $(2, 6)$, $(7, 16)$ and $(9, 18)$.

4. What must be true of a sequence of real numbers a_1, a_2, a_3, a_4, ... if the values of

$$a_1,\ \min(a_1, a_2),\ \min(a_1, a_2, a_3),\ \min(a_1, a_2, a_3, a_4),\ \ldots$$

are all different?

5. Precisely describe the intersection of the graphs of $z = x + 2y + 7$ and $z = 3x + y + 5$.

Hints and answers on the next page. $\Longrightarrow$

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Hints and Answers

1. The graph consists of the line $y = 2x$ for $x \leq 3$, followed by the line $y = x + 3$ for $3 \leq x \leq 7$, followed by the line $y = 10$ for $x \geq 7$. So the graph is a piecewise linear function with three linear components, which meet at the nonlinear points $(3, 6)$ and $(7, 10)$.

2. Using the graph from the previous part, we see that the horizontal line $y = 5$ intersects the graph in the $y = 2x$ component, so when $2x = 5$, or $x = 2.5$. Notice that $x = 2$ is not the answer, since $\min(2 \cdot 2, 2 + 3, 10) = 4$, not 5. The second answer is $x = 6$, while there is no value giving 14, because $\min(2x, x + 3, 10)$ is at most 10.

3. By finding the equations of the lines joining successive points on the graph, we deduce that the linear components have equations $y = 3x$, $y = 2x + 2$, $y = x + 9$ and $y = 18$. Therefore the desired formula is $\min(3x, 2x + 2, x + 9, 18)$.

4. Observe that if $a_2 \geq a_1$ then $\min(a_1, a_2) = a_1$, which would cause the first two values in the list to be equal. So we must have $a_2 < a_1$. In the same manner we deduce that $a_3 < a_2$, and so forth. In other words, the sequence of numbers a_1, a_2, a_3, $\ldots$ must be strictly decreasing.

5. The graphs of $z = x + 2y + 7$ and $z = 3x + y + 5$ will each be a plane in space. The graphs intersect above a point (x, y) precisely when the values of $x + 2y + 7$ and $3x + y + 5$ are equal; so where $x + 2y + 7 = 3x + y + 5$, or $y = 2x - 2$. So the intersection of these planes is a line in space situated over (or under) the line $y = 2x - 2$ in the plane. The height of the line above a point (x, y) on the line $y = 2x - 2$ is the common value of $x + 2y + 7$ and $3x + y + 5$, which can also be written as $5x + 3$. (It is possible to describe this line using parametric equations, but we will not need this technique on the Team Play contest, so we omit such a description here.)