



The Mandelbrot Competition

Round Four Test

Time Limit:
40 minutes

Name: _____

1. Compute the value $\frac{(3\frac{1}{2})(4\frac{1}{3})(5\frac{1}{4}) \cdots (9\frac{1}{8})}{(3\frac{1}{4})(4\frac{1}{5})(5\frac{1}{6}) \cdots (9\frac{1}{10})}$ as a fraction in lowest terms.	1
2. Plot five points in the plane, and connect each pair of points with either a red or blue line segment such that there are five red and five blue segments. Of the ten triangles whose vertices are among the original five points, what is the largest possible number of them whose edges are all the same color?	1
3. A square of side length 8 and a circle with radius 3 have the same center. Let region \mathcal{R} consist of the circle, the square, and the shaded area in between. Eight points are placed in \mathcal{R} such that the smallest distance between two points is $\sqrt{17}$. What is the second smallest distance between any two points?	2
4. Determine $F_{705} - 3F_{701} - 2F_{700}$, writing your answer as F_n for $n > 0$. (Here F_n is a Fibonacci number, where $F_1 = 1$, $F_2 = 1$, and $F_{n+1} = F_n + F_{n-1}$.)	2
5. A box contains two copies of each letter of the alphabet, on 52 cards total. Seven letters are removed, which form the word 'FRIENDS'. Exactly one of these letters is thrown away, then another is taken out of the box to replace it. How many (unordered) sets of letters could possibly result afterwards?	2
6. Find the triple of real numbers x, y, z , none of which equal 0, that make exactly three of these four equations true. $\begin{array}{rcl} 2y & = & x + 4 \\ 3x + 2z + 12 & = & 6y \end{array} \qquad \begin{array}{rcl} 5y + 5 & = & 2x + 2z \\ z + 6 & = & 3y \end{array}$	3
7. The ellipse $\frac{x^2}{100} + \frac{y^2}{19} = 1$ is printed on a 20×20 sheet of paper so that the ellipse is centered and its axes are parallel to the edges of the paper. The sheet of paper is then rolled up into a cylinder so that the two foci of the ellipse are physically touching one other. What is the volume of this cylinder?	3

SCORE: