



★ REGIONAL LEVEL ★

March 2013

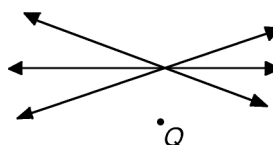
# The Mandelbrot Competition

## Round Five Test

Name: \_\_\_\_\_

Time Limit:  
40 minutes

1. Given three lines that all cross at the same spot and a point  $Q$  below them, how many lines through  $Q$  intersect the rest of the diagram in fewer than three points?



1

2. How many times larger than  $5 \times 10^{20}$  is  $2 \times 10^{50}$ ? Write your answer in the form  $C \times 10^k$  where  $1 \leq C < 10$  and  $k$  is an integer.

1

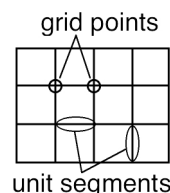
3. Sutton has received three different test scores in his math class. It turns out that the average of his test scores is 6 more than his median (middle) test score. Let  $A$  be the difference between his higher two test scores, and let  $B$  be the difference between his lower two test scores. Determine  $A - B$ .

2

4. Suppose that the graph of  $f(x)$  for  $x \geq 0$  consists of segments starting at  $(0, 0)$ , increasing up to  $(1, 10)$ , dropping directly back down to  $(2, 1)$ , rising linearly up to  $(3, 11)$ , decreasing down to  $(4, 2)$ , and so on, alternating between segments with slope 10 and  $-9$ . For how many values of  $x$  does  $f(x) = 2013$ ?

2

5. Draw a rectangle on a sheet of graph paper whose sides lie along the grid lines. We compute its “score” by awarding 7 points for each grid point enclosed and deducting 3 points for each unit segment inside. The rectangle shown encloses 6 grid points and 17 unit segments, so its score is  $6(7) - 17(3) = -9$ . Suppose that a certain rectangle has height and width both at least 8, and its score is 2013. What is its area?

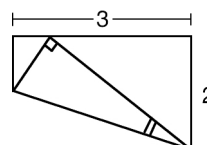


2

6. Laura chooses a positive integer from 1 to 45 at random and lists all of its positive divisors. Then Ben selects one of these divisors, again at random. Which positive integer from 1 to 45 is least likely to occur as Ben’s number?

3

7. A  $2 \times 3$  rectangular sheet of paper is folded over a crease through one corner so that the adjacent corner lands precisely on the opposite side, as shown. Let  $\alpha$  be the measure of the angle marked in the diagram. Calculate  $\tan \alpha$ .



3

SCORE: