



★ NATIONAL LEVEL ★

October 2018

# The Mandelbrot Competition

## Round One Test

Name: \_\_\_\_\_

*Time Limit:*  
40 minutes

1. Let $x$ be the positive real number for which $x^2 + 7x = 35$ . What is the value of $(x + 3)(x + 4)$ ?		①							
2. One can fill in the squares with the digits 1, 3, 6, 6, 7, 8, 9 so that each pair of adjacent digits is relatively prime, meaning they have no common factor other than 1. What is the smallest possible resulting seven-digit number? <div style="text-align: center;"> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> </tr> </table> </div>									①
3. Simplify the product $(\log_{28} 216)(\log_6 2 + \log_6 \sqrt{7})$ , writing your answer in simplest form without logarithms.		②							
4. Selene has 25 different books and 16 different bookmarks. One afternoon Selene loans some of her books and bookmarks to Lillie, who has none. It turns out that now Lillie has exactly one more way of selecting a book and a bookmark (from her items) than Selene does (from the remaining items that she kept). How many ways does Lillie have to choose a book and a bookmark?		②							
5. Let $ABC$ be a triangle with sides of length $AB = 13$ , $BC = 14$ and $AC = 15$ , and let $O$ be its circumcenter. If line $AO$ intersects the circumcircle of $\triangle ABC$ at $N$ , then compute distance $CN$ .		②							
6. Define $r(p, q)$ , for positive integers $p$ and $q$ , to be the remainder when $p$ is divided by $q$ . Find the number of ordered pairs of positive integers $(a, b)$ such that $1 \leq a, b \leq 30$ and $r(a, b) + r(b, a) = a$ .		③							
7. Humpty Dumpty is about to climb up a ladder with seven rungs. At every step (including the first step up to rung one), there is a $\frac{1}{2}$ probability that he reaches the next rung, and a $\frac{1}{2}$ probability that he ends up on the ground again below the first rung. On average, how many steps will it take him to reach rung seven and sit down on the wall?		③							

SCORE:

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