March 2010

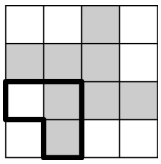
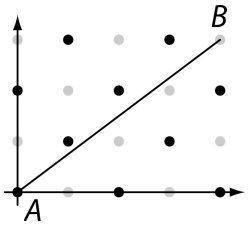
⋆ Regional Level ⋆

The Mandelbrot Competition

Round Five Test

Time Limit: 40 minutes

Name: _____

1. Suppose that $ABCD$ is an arbitrary rectangle. Let M be the midpoint of $\overline{AB}$ and let N be the midpoint of $\overline{AD}$. Then $\overline{MN}$ intersects diagonal $\overline{AC}$ at a point P. Determine the value of the ratio AP/PC.		1
2. Mr. Strump has three daughters with integer ages. One of them is currently twice the age of another. One year from now it will again be the case that one of the girls is twice the age of another. Four years after that the same thing will occur once more. How old is the eldest daughter currently?		1
3. In the Flip puzzle shown at right you use each turn to choose any three squares in the shape of an L, such as the three outlined here, and flip all three of them, from shaded to white or from white to shaded. What is the minimum number of turns needed to make all the squares shaded? 		2
4. How many ways are there to write 84,000 as the product of two relatively prime positive integers? The order of the two factors does not matter; thus $8 \cdot 13$ and $13 \cdot 8$ would not be different ways of writing 104 as a product.		2
5. For a positive integer n, we define $g(n)$ to equal n plus the sum of the digits of n. Thus $g(25) = 25 + 2 + 5 = 32$, for example. Find the smallest positive integer n such that $g(n) = g(N)$ for some larger integer $N > n$.		2
6. Color the points in the plane having integer coordinates alternately red and blue, as shown in the diagram. Then color each point on the segment $\overline{AB}$ from $(0, 0)$ to $(4, 3)$ the same as the nearest colored point, or black if there is a tie. The black points split $\overline{AB}$ into several shorter blue and red segments. How long is the shortest such segment? 		3
7. Let θ be the angle in the first quadrant for which $4\cos(2\theta) + 3\sin(2\theta) = \frac{41}{16}$. Compute the value of $3\cos\theta + \sin\theta$.		3

SCORE: