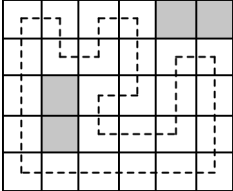


The Mandelbrot Competition

Round One Test

Time Limit:
40 minutes

Name: _____

<p>1. What is the maximum number of 1×2 dominoes one can place on a 5×6 grid such that a connected loop can be drawn through the remaining squares, if no two dominoes overlap or occupy adjacent squares? (The dotted loop may only move horizontally or vertically, not diagonally.)</p> 		1
<p>2. Let $f(x) = ax^2 + bx + c$ be a quadratic with positive integer coefficients. If $f(1) = 21$ and $f(10) = 201$, then what is $f(100)$?</p>		1
<p>3. For how many positive integers b does the base b expansion of π start out as $3.1\dots$? (Note $b = 10$ works, since in base 10 we have $\pi = 3.14159\dots$)</p>		2
<p>4. Let us say that a decade is <i>primeval</i> if it contains four prime numbers. For instance, the decade from 1480 to 1490 was primeval, since 1481, 1483, 1487 and 1489 are all primes. Let p_1, p_2, p_3 and p_4 be the primes (in order) in the next primeval decade after 2020. Compute the value of $p_2p_3 - p_1p_4$.</p>		2
<p>5. A certain collection of numbered index cards includes one card with a 1 written on it, two cards with a 2, and so forth up to n cards showing an n, for some positive integer n. Determine n, if it is the case that drawing a card at random from this collection results in a value of 2017, on average.</p>		2
<p>6. Let $\triangle ABC$ be a triangle with $AB = 13$, $AC = 14$ and $BC = 15$, and let D be the point on \overline{BC} such that $BD = 5$. If E is the foot of the altitude from C to line AD, then find the area of $\triangle CDE$.</p>		3
<p>7. The polynomial $9x^3 + 9x^2 - 1$ has three real roots r, s and t, all between -1 and 1. Calculate the value of $\sum_{k,m,n=0}^{\infty} r^k s^m t^n$, where the sum extends over all ordered triples k, m, n of non-negative integers.</p>		3

SCORE: