February 2010

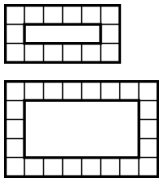
⋆ National Level ⋆

The Mandelbrot Competition

Round Four Test

Time Limit: 40 minutes

Name: _____

1. Three identical fire trucks can together pump 1200 gallons of water in half a minute. How long, in minutes, will it take five such fire trucks to pump a total of 6000 gallons?		1
2. There are exactly two real numbers x satisfying $9^x + 3 = 12x$. Find the smaller of these two numbers.		1
3. A rectangular track is a region consisting of 1×1 squares arranged in a rectangular shape. A 3×6 and 5×8 rectangular track are pictured at right, containing 14 and 22 squares, respectively. What is the total number of unit squares in a 3×6, a 5×8, a 7×10, ..., and a 97×100 rectangular track? 		2
4. There are several ways to translate (shift) the graph of $y = x^2$ so that its vertex lies on the line $y = x - 1$ and the y-intercept is equal in value to one of the x-intercepts. What is the largest possible value of the other x-intercept?		2
5. Find the remainder when the number $1^{40}2^{39}3^{38} \cdots 39^2 40^1$ is divided by 41. (Note that the remainder r will be an integer in the range $0 \leq r \leq 40$.)		2
6. Let G be the centroid of $\triangle ABC$, and let M be the midpoint of side BC. Suppose that $CG = CM$, that $AC = 13$, and that the lengths of sides AB and BC are also integers. Determine the perimeter of $\triangle ABC$.		3
7. At a carnival game you generate a random number between 0 and 2, then a random number between 1 and 3, then a random number between 2 and 4, and so on, for as long as each number is larger than the previous one. If you earn \$1 for each number successfully generated (including the first one), then how many dollars can you expect to receive?		3

SCORE: