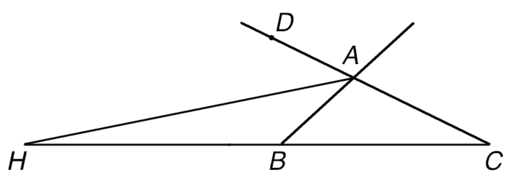
Answer Key	
1. 28	4. 7
2. 2/7	5. 42
3. $4°$	6. 10
	7. 5/12

1. Let b be the number of beads that Rohan and Arjun start with. After the trade Rohan has seven more beads than before, so Arjun has seven less. If Rohan is to have twice as many beads at this point we must have $b + 7 = 2(b - 7)$. Solving this equation gives $b = 21$, meaning that Rohan now has **28** beads.

2. A systematic search reveals that the smallest fraction over 30% is $\frac{1}{3}$, which is about 33.3%, while the largest fraction below 30% is $\frac{2}{7}$, which is approximately 28.6%. Comparing the distance to 30% in each case, we discover that **2/7** comes the closest.

3. Let the angle bisector of angle $\angle DAB$ meet line BC at H, as shown in the diagram below, which is not quite drawn to scale.



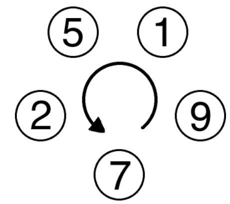
The Exterior Angle Theorem tells us that

$$m\angle DAB = m\angle ABC + m\angle ACB = 20° + 12° = 32°.$$

The angle bisector splits this amount in half, so $m\angle HAB = 16°$. Applying the Exterior Angle Theorem once again to triangle HAB, we find

$$m\angle AHB = m\angle ABC - m\angle HAB = 20° - 16° = \mathbf{4°}.$$

4. Observe that we cannot have have an even, odd, and even digit in this order around the circle. For then the sum of the first two would be odd, which cannot be a multiple of the remaining even number. A similar argument shows that one cannot place two even numbers adjacent to one another, for then the digit before them must also be even, and hence so must the digit before that, and so on back around the circle. But then all the digits would be even, when there are only four available. These two facts together imply that the remaining four digits are odd. From there trial and error is probably the quickest way to finish. The solution shown at left is unique, so the correct answer is **7**.



5. The top left vertex can be any of three colors, while its neighbor to the right can then be any of the two remaining colors. So there are six possible ways to start the coloring; say we color them red and green, from left to right. Next, the middle two vertices (from top to bottom) cannot both be brown, so either one is brown or neither are.

Suppose the right middle vertex is brown. Then the left middle and bottom right vertices must both be green, leaving the bottom left to be either red or brown, giving two colorings. By symmetry there will also be two colorings when the left middle vertex is brown, bringing the total to four. Now suppose that neither of the middle vertices are brown; then the left middle must be green while the right middle is red. This leaves the bottom left to be red or brown, while the bottom right is green or brown. Any combination will work, except when they are both brown, giving three more colorings. Therefore the final total is $6(4 + 3) = \mathbf{42}$.

6. There are many ways to approach this problem; here is a relatively quick solution. Consider the sides of the triangles and squares between the inner and outer borders. Observe that each triangle causes the next such side to be rotated $\frac{\pi}{13}$ radians relative to the previous, while each square results in no rotation, since the sides are parallel. Therefore in order for the sides to wrap completely around the loop we will need a total rotation of 2π radians, which will require 26 triangles. This leaves **10** squares, since there are 36 shapes in total.

7. At first glance it might appear that the answer should be $\frac{1}{2}$, since there ought to be as many instances where the middle number is closer to the smaller number as there are instances where the middle number is closer to the larger one. However, in a few special cases neither scenario occurs—whenever the middle number is precisely in the middle of the smaller and larger numbers. This provides the following shortcut.

We first compute that there are

$$\binom{10}{3} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$$

ways to choose three index cards. It is relatively straight-forward to list the cases in which the middle number is exactly between the others:

$$\{1,2,3\}, \quad \{1,3,5\}, \quad \{1,4,7\}, \quad \{1,5,9\}, \quad \{2,3,4\}, \ldots$$

We count twenty such cases in all, leaving 100 triples in which the middle number is actually closer to one of the others. By symmetry the middle number will be closer to the larger number in half of these cases, giving an answer of $\frac{50}{120} = \mathbf{\frac{5}{12}}$.

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⋆ Regional Level ⋆

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