Answer Key	
1. 6	4. 53
2. B	5. 66
3. 102	6. 24
	7. 39π

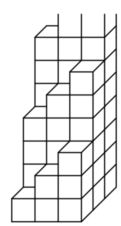
1. Suppose we use the digits 1 and 6 to obtain $1 \times 6 = 6$. Then the only pair of distinct digits left that add to 8 are $3 + 5 = 8$. Once these are used, we can only write $8 - 4 = 4$ to obtain a difference of 4, but then it's impossible to get a quotient of 2 using the remaining digits 2, 7, 9. So writing $1 \times 6 = 6$ was a bad idea.

The other possibility is to write $2 \times 3 = 6$. Now we can only add a pair of distinct digits to get 8 as $1 + 7 = 8$. Next we look for a quotient of 2, which is possible with what's left by dividing $8 \div 4 = 2$. Finally, a difference of 4 is obtained via $9 - 5 = 4$, leaving only **6** left.

2. We are not given the distance from Jack's home to the bean store, so we just select a convenient distance. For instance, suppose the store were 120 feet away. Then the total roundtrip times would be

$$\frac{120}{3} + \frac{120}{8} = 55 \text{ sec}, \qquad \frac{120}{4} + \frac{120}{5} = 54 \text{ sec},$$

meaning the second day was just slightly faster. If the store had been twice as far away, then all the times would double, but this would not change our answer. (Same for any other distance from home to store.) Hence the answer is $\boldsymbol{B}$, regardless of the distance.



3. Observe that the number of unit squares facing forward is exactly $7 + 8 + 9 = 24$. (Imagine a purely front-on perspective.) Obviously the number or rear-facing squares is also 24. Similarly reasoning reveals that the number of left-facing squares is $3 + 6 + 9 = 18$, and same for right-facing squares. Finally, there are 9 squares facing up and another 9 squares facing down, for a total surface area of

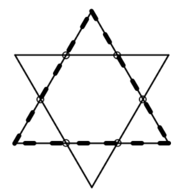
$$24 + 24 + 18 + 18 + 9 + 9 = \mathbf{102}.$$

4. Since the only even prime is 2, the equation $q+r+s = 89$ implies that all three of q, r, s are odd primes. But then we deduce from $p+q+r = 72$ that p must even, hence $p = 2$. Our three equations now reduce to

$$q + r = 70, \quad r + s = 72, \quad q + r + s = 89.$$

Adding the first two and subtracting the third yields $r = 53$, from which we quickly conclude that $q = 17$ and $s = 19$. Therefore $r = \mathbf{53}$ is the largest of the four primes.

5. We first focus on how the segments marked with heavy dotted lines are colored. If they are all the same color (say all green), then each of the



other three segments could be either yellow or orange, giving 8 ways to finish. The same thing occurs if the dotted segments are all yellow, or all orange, leading to 24 colorings so far. On the other hand, if the dotted segments are all different colors (which can occur in 6 ways), then each of the remaining segments only has a single option. This adds 6 more colorings to our list. The reader should analyze the situation in which two of the dotted segments have one color but the third has another color. One finds that there are 18 ways to color the dotted segments, times 2 ways to finish in each case, giving another 36 valid colorings, for a total of $24 + 6 + 36 = \mathbf{66}$ colorings.

6. It will be convenient to let w represent the quantity $\log_x y$. According to Laws of Logarithms we then have $\log_y x = \frac{1}{w}$, meaning that $\log_y(x^2) = 2\log_y x = \frac{2}{w}$. Therefore the first equation gives

$$w - \frac{2}{w} = 1 \quad \Longrightarrow \quad w^2 - w - 2 = (w+1)(w-2) = 0,$$

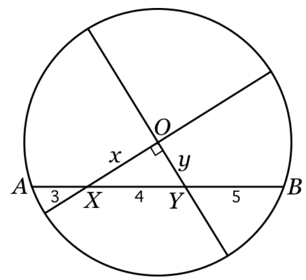
hence $w = -1$ or 2. The first case leads to $\log_x y = -1$, or $y = x^{-1} = \frac{1}{x}$. Hence the second equation gives $x + \frac{1}{x} = 20$, or $x^2 - 20x + 1 = 0$. This quadratic has two positive roots whose sum is 20, by Viete. The other possibility is that $w = 2$, which corresponds to $\log_x y = 2$, or $y = x^2$. Substituting into the second equation yields $x^2 + x = 20$, which has the roots $x = 4$ and $x = -5$. Only the first of these is positive, however, so the desired sum of all possible positive values of x is $20 + 4 = \mathbf{24}$.

7. Label the distances from X and Y to the center O as x and y, and let r be the radius of the circle. Then by power of a point at X we have

$$(r - x)(r + x) = (3)(4 + 5) \implies r^2 - x^2 = 27.$$

Doing the same thing at Y gives

$$r^2 - y^2 = (5)(3 + 4) = 35.$$



Adding these equations, and using the fact that $x^2 + y^2 = 16$ by Pythagorean, we find that $2r^2 - 16 = 62$, so $r^2 = \frac{1}{2}(78) = 39$. Thus the area of the circle is $\pi r^2 = \mathbf{39}\pi$.

Problem Credits

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