



★ REGIONAL LEVEL ★

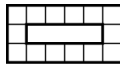

February 2010

# The Mandelbrot Competition

## Round Four Test

Name: \_\_\_\_\_

*Time Limit:*  
40 minutes

1. Three identical fire trucks can together pump 1200 gallons of water in half a minute. How long, in minutes, will it take five such fire trucks to pump a total of 6000 gallons?		①
2. Al, Betty and Cate each have ten marbles. On the first turn, Al takes a single marble from one of the others. On the second turn Betty takes two marbles from one of the others. Then Cate takes three marbles from one of the others, Al takes four marbles, and so on around. What is the minimum number of turns required for all players to once again have ten marbles each?		①
3. There are exactly two real numbers $x$ satisfying $9^x + 3 = 12x$ . Find the smaller of these two numbers.		②
4. A rectangular track is a region consisting of $1 \times 1$ squares arranged in a rectangular shape. A $3 \times 6$ and $5 \times 8$ rectangular track are pictured at right, containing 14 and 22 squares, respectively. What is the total number of unit squares in a $3 \times 6$ , a $5 \times 8$ , a $7 \times 10$ , ..., and a $97 \times 100$ rectangular track?	 	②
5. Let $D$ be the point on side $\overline{BC}$ of triangle $ABC$ for which $\overline{AD}$ is the angle bisector of $\angle A$ . If $AB = AD = 6$ and $CD = 2(BD)$ , then find length $BD$ .		②
6. There are several ways to translate (shift) the graph of $y = x^2$ so that its vertex lies on the line $y = x - 1$ and the $y$ -intercept is equal in value to one of the $x$ -intercepts. What is the largest possible value of the other $x$ -intercept?		③
7. Find the remainder when the number $1^{40}2^{39}3^{38} \cdots 39^240^1$ is divided by 41. (Note that the remainder $r$ will be an integer in the range $0 \leq r \leq 40$ .)		③

SCORE: