



★ NATIONAL LEVEL ★

December 2017

# The Mandelbrot Competition

## Round Three Test

Name: \_\_\_\_\_

Time Limit:  
40 minutes

1. One can fill in the squares with the digits from 1 to 7 so that each digit appears exactly once and each pair of adjacent digits, viewed as a two-digit number, is divisible by 7 or 9. What is the resulting seven-digit number?

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①

2. Four points  $A, B, C, D$  in the plane are situated so that  $AB = 4$ ,  $BC = 5$ ,  $CD = 6$  and  $DA = 8$ . What is the minimum possible value of distance  $AC$ ?

①

3. Oliver rolls a fair die and flips a fair coin. If the coin comes up heads, he multiplies the number on the die by 2. Otherwise, he multiplies the number on the die by 3. What is the probability that the result is divisible by 6?

②

4. We say that a positive integer  $m$  is *fortunate* if there are three distinct primes larger than 4 such that dividing  $m$  by each of these three primes gives a remainder of 4. Determine the smallest multiple of 3 that is fortunate.

②

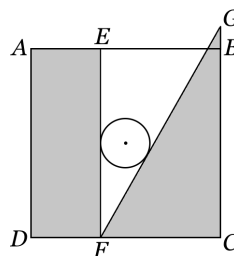
5. Consider the function  $f(x) = x^2 + 20x + 17$ . For what value of  $a$  are both the solutions to  $f(x) = x$  also solutions to  $a - f(a - f(x)) = x$ ?

②

6. Find the only two-digit number  $b$  such that  $b^{47}$  ends with the digits 47.

③

7. Suppose square  $ABCD$  with side length 2 and a circle with radius  $r < 1$  have the same center. Let  $E$  and  $F$  be points on  $\overline{AB}$  and  $\overline{CD}$  such that  $\overline{AD} \parallel \overline{EF}$  and such that  $\overline{EF}$  is tangent to the circle. Next let  $G$  be the point on line  $BC$  such that  $\overline{FG}$  is tangent to the circle. Find the value of  $r$  for which the areas of  $ADFE$  and  $CFG$  are equal.



③

SCORE:

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