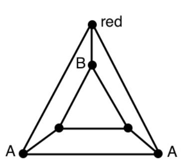
Answer Key	
1. Neeyanth	4. 20/7
2. 1057	5. 63
3. 24	6. 66336
	7. $y = 0.7x - 14.5$

1. At any given point in time, a person not holding the Frisbee has one throw for each catch (like Veer), unless they began with the Frisbee, in which case they have one more throw than catch (like Joshua). However, Neeyanth does not fall into either of these categories, so **Neeyanth** must be the one currently holding the Frisbee.

2. Notice that in any sequence of terms that steadily increases (called an *arithmetic progression*), the middle term must be exactly halfway between the first and last, since we add the common difference the same number of times to get from the first to middle term as we do to get from the middle to last term. Hence the middle term is the average of the first and last, or $\frac{1}{2}(100 + 2014) = \mathbf{1057}$.

3. To begin, let's assume that the topmost dot is red. The other red dot must be at a vertex labeled A or B in order for the red dots to be joined by a segment. If it's at either dot labeled A, then there are two ways to finish the coloring from there, while if it's at dot B then there are four ways to finish the coloring, for a total of 8 valid colorings so far. The same logic applies if the topmost dot is blue or green, giving a grand total of **24** ways to color the dots.



4. Let the radius of the inner and outer circles be r and R. The shaded area is the difference in area of the two circles, so $\pi R^2 - \pi r^2 = 20\pi$. Meanwhile the total boundary is the sum of the two circumferences, meaning that $2\pi R + 2\pi r = 14\pi$. Dividing each equality by π gives

$$R^2 - r^2 = 20, \qquad 2R + 2r = 14.$$

Factoring and dividing by 2 leads to

$$(R + r)(R - r) = 20, \qquad R + r = 7.$$

The width of the ring is $R - r$, so we divide these equalities to obtain $R - r = \mathbf{20/7}$, or $\mathbf{2\frac{6}{7}}$.

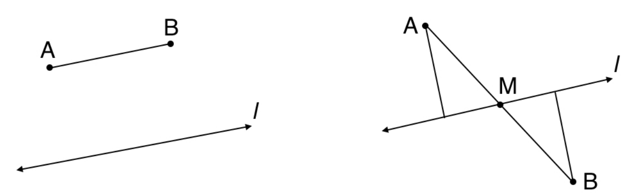
5. We call Shivani's favorite integer x, and the classroom number c. According to Shivani, it is the case that

$$14(x + 1) - c = x^2.$$

Stuart knows the room number c, but doesn't know the value of x, so he must solve the quadratic equation $x^2 - 14x + (c - 14)$. Ordinarily a quadratic would have two solutions, so Stuart would not have enough information to guess Shivani's integer. But there is an exception; namely, when there is a double root. This occurs only if the quadratic has the form $(x - 7)^2 = 0$, or $x^2 - 14x + 49 = 0$. Hence we must have $c = \mathbf{63}$.

6. Clearly the final digit must be a 6, since any power of 2 will be even. Recalling our divisibility test for 4, (that the final two digits, viewed as a two-digit number, must be a multiple of 4) we can further deduce that our power of 2 must end 36, since 66 is not a multiple of 4. In fact, since our power of 2 is divisible by 8, the final three digits must be also, which implies that our number ends 336. Continuing in this manner, we realize that the final four and five digits must be multiples of 16 and 32, respectively, bringing us to 6336 and finally **66336**.

7. The key to an efficient solution is to take a closer look at the geometry of the situation. As the diagram below suggests, if two points A and B



are the same distance from a line, then either the line is parallel to $\overline{AB}$, when the points are on the same side of the line, or else the line passes through the midpoint of $\overline{AB}$, when the points are on opposite sides of the line. (The reader should provide a proof of these claims.) Applying this fact to our diagram, we see that the desired line must pass through precisely two of the midpoints of the sides of $\triangle ABC$, giving three possible positions for the line. But two of these lines have negative slope, ruling them out. The line we seek passes through the midpoints of $\overline{AC}$ and $\overline{BC}$, which have coordinates $(15, -4)$ and $(25, 3)$. Hence its slope is $7/10$, so it has the form $y = 0.7x + b$. Plugging in $x = 15$, $y = -4$ yields $b = -14.5$, so our final answer is $\mathbf{y = 0.7x - 14.5}$.

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⋆ Regional Level ⋆

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Round Three Solutions

Greater Testing Concepts
PO Box 760
Potsdam, NY 13676

The Mandelbrot Competition
www.mandelbrot.org
info@mandelbrot.org