



★ REGIONAL LEVEL ★

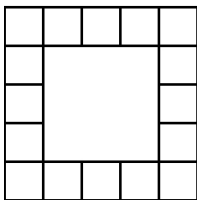
December 2013

The Mandelbrot Competition

Round Two Test

Name: _____

Time Limit:
40 minutes

1. In a certain math class, two-thirds of the students are girls. Suppose that one-quarter of the girls are asleep, while two-fifths of the boys are asleep. What fraction of the class is awake?		1
2. Sixteen squares are arranged into four overlapping lines, with five squares in each line, as shown at right. Suppose we place thirteen chips on the squares so that each line contains exactly five chips. (It is OK to place more than one chip per square.) How many chips must be placed on the four corner squares in total?		1
3. Note that $x = 1$ satisfies the equation $\frac{1}{x+1} - \frac{1}{x+2} = \frac{1}{x+5}$. There is also a negative number satisfying this equation. What is its value?		2
4. Suppose that $P_1P_2P_3 \cdots P_{20}$ is a regular 20-sided polygon. Determine the measure of the acute angle between lines $P_{20}P_{13}$ and $P_{19}P_7$, in degrees.		2
5. A square is sliced into 1000 congruent rectangles using 999 horizontal lines. If each rectangle has a perimeter of 2013, then what is the perimeter of the original square, rounded to the nearest hundred?		2
6. Draw forty-two points equally spaced around the circumference of a circle. How many ways are there to choose three of these points so that the triangle having these points as vertices is isosceles? (Note that an isosceles triangle has at least two sides of equal length.)		3
7. Find all pairs (a, b) of natural numbers with $a < b$ having the property that there exists a right triangle with legs of length a and b whose hypotenuse has length $\frac{1}{3}ab - a - b$.		3

SCORE: