December 2012

⋆ National Level ⋆

The Mandelbrot Competition

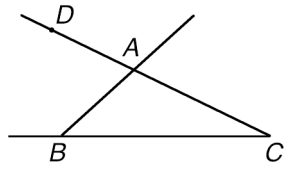
Round Two Test

Time Limit: 40 minutes

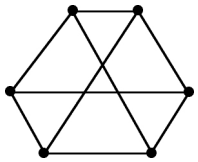
Name: _____

1. Rohan and Arjun have the same number of colored beads. Rohan trades three of his gold beads for ten of Arjun's orange beads, at which point he has twice as many beads as Arjun. How many beads does Rohan now have? (1)

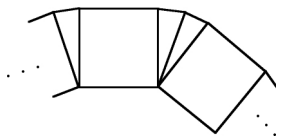
2. Suppose that in the diagram shown at right we have $m\angle ABC = 20°$ and $m\angle ACB = 12°$. Compute the acute angle of intersection formed by the angle bisector of angle $\angle DAB$ and line BC. (1)



3. Each of the six points in this network is connected to its neighbors as well as to the point directly across from it. In how many ways can one color each point either green, red, or brown so that connected points always have different colors? (2)

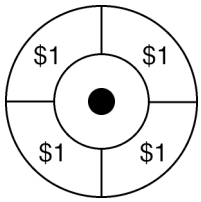


4. Sophie has unit squares and isosceles triangles with two sides of length 1 and a vertex angle measuring $\frac{\pi}{13}$ radians. By arranging them in some order (in the manner shown) she creates a loop whose outer perimeter is a 36-sided polygon. How many squares does she use? (2)



5. Christine numbers ten index cards from 1 to 10, then chooses three cards at random and orders them from smallest to largest. What is the probability that the middle number is closer to the larger number than the smaller one? (2)

6. The game of Collect the Cash begins with a marker in the center. On each turn the marker moves to a neighboring region, with all possible moves equally likely, and the player collects any cash there. (Collected cash is not replaced.) The game is over once the marker returns to the center circle. On average, how much cash will the player collect? (3)



7. Denote the roots of $x^2 + 2012x - 1776$ by α and β, and let $x^2 + Bx + C$ have roots α^{101} and β^{101}. Determine the remainder when B is divided by 101. (3)

SCORE: