



★ NATIONAL LEVEL ★

March 2012

The Mandelbrot Competition

Round Five Test

Name: _____

Time Limit:
40 minutes

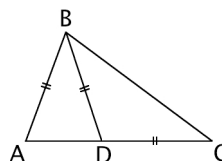
1. How many ways are there to split a group of eight people labeled A, B, C, D, E, F, G , and H into two groups of four people each if A and B must be in the same group but C and D must be in opposite groups?

1

2. Find the real number x satisfying the equation $\frac{6}{5 + \frac{4}{3+x}} = 2$.

1

3. Point D is situated on side \overline{AC} of triangle ABC so that $AB = BD = DC$. If $m\angle ABC = 69^\circ$, then determine the measure of $\angle BAC$.



2

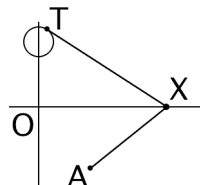
4. A jar contains between 7 and 70 marbles, some of which are orange. The probability of choosing a pair of marbles at random from the jar and obtaining two orange marbles is exactly $1/5$. How many marbles are there in the jar?

2

5. For how many complex numbers z does $x^2 + xz^5 + 2x + z^5 - 5$ have a pair of repeated roots for x ? For instance, when $z = -1$ we obtain $x^2 - x - 6$. This quadratic has roots $x = -2$ and $x = 3$, which are distinct roots, so $z = -1$ is not one of the desired values.

2

6. Given the points $O(0, 0)$ and $A(5, -7)$ and the circle of radius 3 centered at $(0, 7)$, there exists a point T on the circle and a point X on the positive x -axis such that \overline{XT} is tangent to the circle and $\angle TXO \cong \angle AXO$. Determine the x -coordinate of the point X for which this occurs.



3

7. For positive real numbers x let $f(x)$ be the distance from x to the nearest real number all of whose decimal digits are even. Thus $f(1) = 1 - 0.88\overline{8} = \frac{1}{9}$. Find the area between the graph of $f(x)$ and the x -axis for $0 \leq x \leq 10$.

3

SCORE: