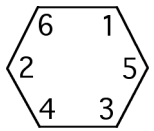
Answer Key

1. $0.23
2. A
3. 58
4. $m = 9,\ n = 7$
5. $(2, 17, 18)$
6. 1458
7. -4.5

1. The instinctive approach, which is to simply compute 5% of \$4.60, turns out to be correct. This is because if the meal costs M dollars, then the original tax will come to $(.05)M$ dollars. After ordering the banana split, the new amount of tax will be $(.05)(M + 4.60) = (.05)M + 0.23$, which is **\$0.23** more than before. (Writing "23 cents" is also acceptable.)

2. An equivalent way to phrase the question is to ask for which of the three graphs it is possible for a line to intersect the graph in three different points. Considered this way, it becomes obvious that this is possible for a sine curve but not for a parabola or natural log graph. Hence the answer is ***A.***

3. In order to obtain the smallest possible hexagon sum it makes sense to keep the numbers 4, 5, and 6 separated around the hexagon so that these numbers are never multiplied together. Furthermore, one should surround the 6 by the 1 and 2 to avoid large multiples of the largest number. Testing the two possible cases, one discovers that placing the 1 next to the 5 gives a smaller hexagon sum, as shown. Hence the minimal value is $6 \cdot 1 + 1 \cdot 5 + 5 \cdot 3 + 3 \cdot 4 + 4 \cdot 2 + 2 \cdot 6 = \mathbf{58}$.

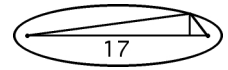


4. We rewrite the equation algebraically to obtain

$$\frac{m-2n}{3m-4n} = 5 \qquad \Longrightarrow \qquad m - 2n = 5(3m - 4n) = 15m - 20n.$$

Moving all terms involving m and n to separate sides of the equation then yields $18n = 14m$, or $9n = 7m$. It is now clear that we may take $\boldsymbol{m = 9}$ and $\boldsymbol{n = 7}$ as a solution. Any positive integer multiple of these numbers is also acceptable, such as $m = 18$ and $n = 14$, for instance.

5. Imagine that the side of length 17 is the fixed base of the triangle. We must locate the third vertex in such a way that the remaining two sides of the triangle add up to 20. By definition, the set of points in the plane with this property is an ellipse, such as the one shown at right. Clearly the closer this third vertex is to one of the first two, the lower it will be relative to the base. In other words, the vertical height will be smaller, leading to a smaller area. Since the side lengths are positive integers, we first try $(1, 17, 19)$, but this triple does not produce a triangle, because $19 > 1 + 17$. The next possibility is **(2, 17, 18)**, which gives the triangle of minimal area. (Answers with the numbers 2, 17, and 18 listed in any order are acceptable.)



6. There are six ways that Troy can place cars in the uppermost two spots; any of the three colors of car may park on the left side, leaving two remaining colors for the car on the right. Once a row is filled, there are exactly three ways to park two cars in the row beneath. For example, if the top row has cars with colors WB, then the next row could be BM, BW, or MW. So there are six ways to fill the first row, then three ways to fill the second row, then three ways to fill the third row, and so on, for a total of

$$6 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = \mathbf{1458}$$

parking arrangements. By the way, did you catch on to the fact that Troy sells vehicles produced by the Bavarian Motor Works (BMW)?

7. We can add the two given fractions algebraically to obtain

$$\frac{A(x-1)}{(x+3)(x-1)} + \frac{6x}{(x+3)(x-1)} = \frac{6x + Ax - A}{(x+3)(x-1)}.$$

In order for the result to reduce to the form $B/(x-1)$, a factor of $(x+3)$ must cancel from the numerator. In other words, $6x + Ax - A$ must be a multiple of $(x+3)$. One way to check for this is to plug in $x = -3$ and

require that the result vanish. Thus we need to have $-18 - 3A - A = 0$, which gives $A = \mathbf{-4.5}$. Sure enough, one can check that adding

$$\frac{-4.5}{x+3} + \frac{6x}{x^2+2x-3} = \frac{1.5}{x-1}.$$

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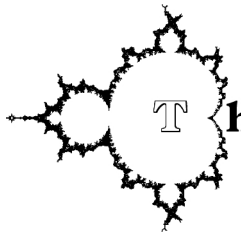

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