December 2008

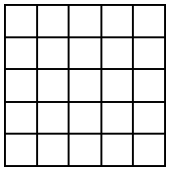
⋆ National Level ⋆

The Mandelbrot Competition

Round Two Test

Time Limit: 40 minutes

Name: _____

1. What is the smallest number of squares that must be shaded in the 5×5 grid at right so that each row and column has at least one shaded square and every pair of adjacent rows and every pair of adjacent columns has a *different* number of shaded squares? (Two rows or columns may have the same number of shaded squares, as long as they are not adjacent.) 	(1)
2. Find the positive real number x for which $5\sqrt{1+x} + 5\sqrt{1-x} = 7\sqrt{2}$.	(1)
3. Austin currently owns some shirts, pants, and pairs of shoes; he chooses one of each to create an outfit. If he were to obtain one more shirt, his total number of outfits would increase by 48. Similarly, if he bought another pair of pants he would have 90 more outfits, while an extra pair of shoes would result in 120 more outfits. How many outfits can he currently create?	(2)
4. Label points $O(0,0)$, $A(1,0)$, $B(1,1)$, $C(2,2)$, and $D(3,0)$ in the plane. If we choose a point P at random along segment $\overline{AB}$, what is the probability that line OP will be closer to C than to D?	(2)
5. The function $f(x) = x+1$ generates the sequence 1, 2, 3, 4, ... in the sense that plugging any number in the sequence into $f(x)$ gives the next number in the sequence. What function $g(x)$ generates the sequence $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, ... in this manner? Write your answer in the form $g(x) = a/(bx+c)$.	(2)
6. Florence draws 100 congruent circles in the plane, all passing through a fixed point P. What is the largest number of regions into which these circles can split the plane? (Include the region outside the circles in your count.)	(3)
7. There is a unique complex number α in the upper half-plane (meaning that $\alpha = a + bi$ with $b > 0$) with the property that the distance from α^2 to 1 is twice the distance from α to 1, while the distance from α^4 to 1 is four times the distance from α to 1. Determine this number α.	(3)

SCORE: