January 2009

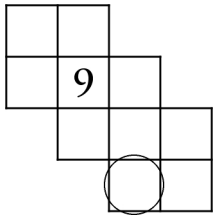
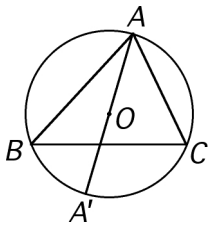
⋆ National Level ⋆

The Mandelbrot Competition

Round Three Test

Time Limit: 40 minutes

Name: _____

1. The value of $\frac{n}{2} + \frac{18}{n}$ is smallest for which positive integer n?	1
2. Find the decimal number between 1 and 10 with the property that moving the decimal point one digit to the left and writing a 3 in front yields a number that is half its original value. (For instance, 4.7 would become 3.47.)	1
3. It is possible to place the digits 0, 1, 2, ..., 9 into the squares at right so that there is one digit per square, each horizontal number is divisible by 7, each vertical number is divisible by 9, no number begins with a 0, and the 9 is placed as shown. Which digit must appear in the circle? 	2
4. Consider a cube on a table with each of the faces painted a different color. Suppose one were to pick up the cube, turn it around, then return it to its original location with the colors permuted. Of the 23 ways to reposition the cube, in how many cases does no color occupy its original position?	2
5. Determine the smallest positive integer m such that $m^2 + 7m + 89$ is a multiple of 77.	2
6. Let $f(x)$ be a function defined for all positive real numbers satisfying the conditions $f(x) > 0$ for all $x > 0$ and $f(x-y) = \sqrt{f(xy)+1}$ for all $x > y > 0$. Determine $f(2009)$.	3
7. Triangle ABC has sides of length $AB = \sqrt{41}$, $AC = 5$, and $BC = 8$. Let O be the center of the circumcircle of $\triangle ABC$, and let A' be the point diametrically opposite A, as shown. Determine the area of $\triangle A'BC$. 	3

SCORE: _____