February 2009

⋆ National Level ⋆

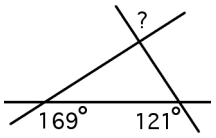
The Mandelbrot Competition

Round Four Test

Time Limit: 40 minutes

Name: _____

1. Three intersecting lines form three sets of angles, as shown to the right. If the two marked angles measure 169° and 121°, then compute the measure of the third angle, indicated by the '?' (question mark).



(1)

2. Carolina, Dakota, and Montana stand facing one another. In how many ways can they perform three simultaneous handshakes if no person shakes their own hand? Note that it matters which hand is used: Montana shaking Dakota's left hand is different from her shaking his right hand.

(1)

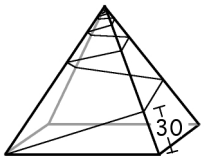
3. Let α and β be complex numbers such that $\alpha + \beta$ and $i(\alpha - 2\beta)$ are both positive real numbers. If $\beta = 3 + 2i$, then compute α.

(2)

4. Jordan is eating green, purple, and yellow jellybeans. He is equally likely to prefer green jellybeans the most, second most, or least, and he says that he likes purple ones more than yellow. What is the probability that he also favors green jellybeans over yellow ones?

(2)

5. A pyramid has a square base, triangular sides, and eight edges that are each 80 meters long. A straight path begins at one corner of the square base, slanting upwards to meet the next edge at a point 30 meters along that edge from the corner, as shown. The path continues around the pyramid, always slanting upward by the same amount, making infinitely many turns. What is the total length of the path?



(2)

6. Lattice points are points (a, b) in the Cartesian plane both of whose coordinates are integers. If a square has all four vertices at lattice points and has area 2009, then how many lattice points are in the interior of the square?

(3)

7. Let a, b, and c be the unique trio of positive integers with $a < b < c$ that satisfy the equations $a + b + c = 79$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{5}$. Determine c.

(3)

SCORE: